Distributed Turbo Coding With Soft Information Relaying in Multihop Relay Networks

Yonghui Li, *Member, IEEE*, Branka Vucetic, *Fellow, IEEE*, Tan F. Wong, *Senior Member, IEEE*, and Mischa Dohler, *Member, IEEE*

Abstract—It has been shown that distributed turbo coding (DTC) can approach the capacity of a wireless relay network. In the existing DTC schemes, it is usually assumed that error-free decoding is performed at a relay. We refer to this type of DTC schemes as perfect DTC. In this paper, we propose a novel DTC scheme. For the proposed scheme, instead of making a decision on the transmitted information symbols at the relay as in perfect DTC, we calculate and forward the corresponding soft information. We derive parity symbol soft estimates for the interleaved source information when only the *a posteriori* probabilities of the information symbols are known. The results show that the proposed scheme can effectively mitigate error propagation due to erroneous decoding at the relay. Simulation results also confirm that the proposed scheme approaches the outage probability bound of a distributed two-hop relay network at high signal-to-noise ratios.

Index Terms—Cooperative diversity, distributed coding, multihop transmission, relay network, soft information relaying, turbo coding, wireless sensor network.

I. INTRODUCTION

IN RELAY NETWORKS, signals are transmitted from one terminal to another through a number of relays. The main advantage of doing so is a reduced signal transmit power [1]–[5]. Recently, this concept has been applied in cooperative wireless systems [6], [7], where each mobile terminal receives signals from other mobiles, and then relays them to the destination.

To improve the performance in a relay network, some cooperative and distributed coding schemes [8]–[13] have been developed recently. A distributed turbo coded (DTC) system is developed for a two-hop relay network in [11]–[13]. The source broadcasts the coded signals to both the destination and relay. The relay then decodes the received signals, and interleaves them prior to the encoding. The signals received at the destination consist of coded information symbols transmitted from the source and coded interleaved information symbols transmitted from the relay. These two signals form a distributed turbo code [25]. It has been shown that such a coding strategy performs close to the theoretic outage probability bound of a relay channel
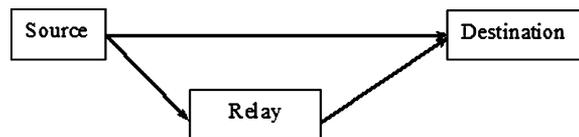
Fig. 1. Block diagram of a two-hop relay system with a direct link.

[11]. However, in the existing DTC schemes, it is usually assumed that the relay can perform error-free decoding. We refer to such DTC schemes as perfect DTC. This scenario can be realized by automatic repeat request (ARQ) in the link from the source to the relay. However, the use of ARQ will reduce the system transmission throughput.

In this paper, we propose a distributed turbo coding scheme with soft information relaying (DTC-SIR). In the proposed scheme, instead of making decision on the transmitted information symbols at the relay, as in [11]–[13], the relay calculates and forwards the soft estimates of the transmitted symbols. The relay first calculates the *a posteriori* probabilities (APPs) of the information symbols. Then, it calculates the parity symbol soft estimates of the interleaved source information based on the APP of the information symbols. Analytical results show that the proposed scheme approaches the performance of the perfect DTC schemes at a high signal-to-noise ratio (SNR) and achieves a higher throughput than latter.

The rest of this paper is organized as follows. Section II introduces the system and channel models. Section III presents the proposed DTC-SIR scheme. Performance analysis and calculation of outage probability bounds are carried out in Sections IV and V, respectively. In Section VI, we present simulation results. Conclusions are drawn in Section VII.

II. SYSTEM MODEL

For simplicity, we consider a two-hop relay system, consisting of one source, one relay and one destination. Fig. 1 gives a block diagram of the two-hop relay system with a direct link from the source to the destination.

A block diagram of a parallel concatenated DTC system is shown in Fig. 2. The transmitted source information binary stream, denoted by $\mathbf{B}$, is represented by

$$\mathbf{B} = (b_1, \ldots, b_k, \ldots, b_l) \tag{1}$$

where b_k is the binary symbol transmitted at time k and l is the frame length.

The binary information sequence $\mathbf{B}$ is first encoded by a channel encoder. For simplicity, we consider a recursive
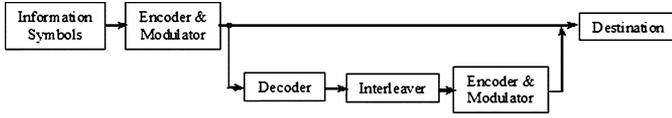
Fig. 2. A parallel concatenated DTC for the relay system in Fig. 1.

Fig. 3. Block diagram of DTC-SIR.

systematic convolutional code (RSCC) [25] with a code rate of $1/2$.

Let $\mathbf{C}$ represent the corresponding codeword, given by

$$\mathbf{C} = (\mathbf{C}_1, \ldots, \mathbf{C}_k, \ldots, \mathbf{C}_l) \tag{2}$$

where $\mathbf{C}_k = (b_k, c_k)$ is the codeword of $b_k, c_k \in \{0,1\}$, b_k is the information symbol and c_k is the corresponding parity symbol.

The binary symbol stream $\mathbf{C}$ is then mapped into a modulated signal stream $\mathbf{S}$. For simplicity, we consider a binary phase-shift keying (BPSK) modulation. The modulated codeword, denoted by $\mathbf{S}$, is given by

$$\mathbf{S} = (\mathbf{S}_1, \ldots, \mathbf{S}_k, \ldots, \mathbf{S}_l) \tag{3}$$

where $\mathbf{S}_k = (s_k^I, s_k^P)$, $s_k^I, s_k^P \in \{-1, 1\}$, are the modulated information and parity signals transmitted by the source at time $2k-1$ and $2k$, respectively.

We assume that the source and relay transmit data through orthogonal channels [5]–[12]. For simplicity, we will concentrate on a time division multiplex [8]–[12], for which the source and relay transmit in separate time slots. The proposed scheme can be extended to other orthogonal transmission techniques in a straightforward manner.

The source first broadcasts the information to both the destination and relay. The received signals at the relay and destination, at time $2k-1$ and $2k$, denoted by $\mathbf{y}_{\text{sr},k}$ and $\mathbf{y}_{\text{sd},k}$, respectively, can be expressed as

$$\mathbf{y}_{\text{sr},k} = \sqrt{P_{\text{sr}}} h_{\text{sr}} \mathbf{S}_k + \mathbf{n}_{\text{sr},k} \tag{4}$$

$$\mathbf{y}_{\text{sd},k} = \sqrt{P_{\text{sd}}} h_{\text{sd}} \mathbf{S}_k + \mathbf{n}_{\text{sd},k} \tag{5}$$

where $\mathbf{y}_{\text{sr},k} = (y_{\text{sr},k}^I, y_{\text{sr},k}^P), \mathbf{y}_{\text{sd},k} = (y_{\text{sd},k}^I, y_{\text{sd},k}^P), P_{\text{sr}} = P_s \cdot (G_{\text{sr}})^2, P_{\text{sd}} = P_s \cdot (G_{\text{sd}})^2$ are the received signal power[1] at the relay and destination, respectively, P_s is the source transmit power, and $G_{\text{sr}} = ((\lambda_c)/(4\pi d_0))((d_{\text{sr}})/(d_0))^{-\kappa/2}, G_{\text{sd}} = ((\lambda_c)/(4\pi d_0))((d_{\text{sd}})/(d_0))^{-\kappa/2}$ [21], [22] are the channel gains between the source and relay and that between the source and destination, respectively, d_{sr} and d_{sd} are the distances between the source and relay, and between the source and destination, respectively, d_0 is a reference distance, λ_c is the carrier wavelength and κ is a path loss factor with values typically in the range $2 \leq \kappa \leq 6$. Also, h_{sr} and h_{sd} are the fading coefficients between the source and relay and between the source and destination, respectively. They are modeled as zero-mean, independent circular symmetric complex Gaussian random variables. In this paper, we consider a quasi-static fading

[1]Note that the gain is usually related to the power; however, for notational simplicity of subsequent exposure, we have opted to relate them to the amplitude, leading to the square of the gain.
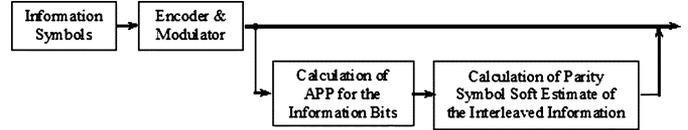
channel, for which the fading coefficients are constant within one frame and change independently from one frame to another. Furthermore, $\mathbf{n}_{\text{sr},k} = (n_{\text{sr},k}^I, n_{\text{sr},k}^P), \mathbf{n}_{\text{sd},k} = (n_{\text{sd},k}^I, n_{\text{sd},k}^P)$ and they are zero-mean complex Gaussian random variables with two-sided power spectral density of $N_0/2$ per dimension. In this paper, we assume that all noise processes have the same variances, without loss of generality. Different noise variances can be taken into account by appropriately adjusting the channel gain.

In the standard DTC, the relay decodes the received signals from the source, interleaves the decoded information symbols, re-encodes and sends them to the destination. Let $x_{r,k}$ represent the signal transmitted from the relay at time k. It satisfies the following transmit power constraint

$$E(|x_{r,k}|^2) \leq P_2 \tag{6}$$

where P_2 is the transmitted power limit at the relay.

The corresponding received signal at the destination at time k, denoted by $y_{\text{rd},k}$, can be written as

$$y_{\text{rd},k} = G_{\text{rd}} h_{\text{rd}} x_{r,k} + n_{\text{rd},k} \tag{7}$$

where G_{rd} is the channel gain between the relay and destination, h_{rd} is the Gaussian complex fading coefficient between the relay and destination, and $n_{\text{rd},k}$ is a zero-mean complex Gaussian random variable with two-sided power spectral density of $N_0/2$ per dimension. The overall received signals at the destination consist of the coded source signal transmitted from the source given in (5) and the coded interleaved source signal transmitted from the relay given in (7). These two signals form a standard distributed turbo code.

III. DISTRIBUTED TURBO CODING WITH SOFT INFORMATION RELAYING (DTC-SIR)

In this section, we propose a DTC scheme with soft information relaying when imperfect decoding occurs at the relay. Rather than making a decision on the transmitted symbol at the relay as in the conventional schemes, we calculate the soft information of the received signals. The challenge is the derivation of the parity symbol soft estimates for the interleaved source information when only the APPs of the information symbols are known. We derive an expression for the parity symbol soft estimates based on the APPs of information symbols.

The block diagram of the DTC-SIR system is shown in Fig. 3. In the proposed scheme, the processing at the relay can be divided into two steps. The decoder first uses a maximum *a posteriori* probability (MAP) decoding algorithm to calculate the APPs of the information symbols. At the second step, the relay utilizes the derived APPs to calculate the parity symbol soft estimates for the interleaved information symbols.

A. Calculation of the APPs for the Information Symbols

Let $\mathbf{y}_{\mathrm{sr}} = (\mathbf{y}_{\mathrm{sr},1}, \ldots, \mathbf{y}_{\mathrm{sr},l})$ represent the received signal sequence at the relay, where $\mathbf{y}_{\mathrm{sr},k}$ are given in (4).

The decoder is based on a BCJR MAP decoding algorithm [16], [25]. The relay uses $\mathbf{y}_{\mathrm{sr}}$ to calculate the APP of $b_k, k = 1, \ldots, l$, denoted by $P(b_k = w \,|\, \mathbf{y}_{\mathrm{sr}}), w \in \{0, 1\}$ [16]

$$
\begin{aligned}
&P(b_k = w \,|\, \mathbf{y}_{\mathrm{sr}}) \\
&= h \sum_{m,m'=0, b(k)=w}^{m,m'=M_s-1} \alpha_{k-1}(m')\beta_k(m)\gamma_k(m,m')
\end{aligned}
$$

where h is a constant such that $\sum_w P(b_k = w | \mathbf{y}_{\mathrm{sr}}) = 1$

$$
\gamma_k(m,m') = \exp\left(-\frac{\|\mathbf{y}_{\mathrm{sr},k} - \sqrt{P_{\mathrm{sr}}} h_{\mathrm{sr}} \mathbf{s}_k\|^2}{N_0}\right) \qquad (8)
$$

where $\|x\|$ is the Frobenius norm of x, m', and m are a pair of states connected with $b_k = w$ in the trellis, M_s is the number of states in the trellis, $\alpha_i(m')$ and $\beta_i(m)$ are the feed-forward and the feedback recursive variables, which can be calculated as

$$
\begin{aligned}
\alpha_i(m') &= \sum_m \alpha_{i-1}(m)\gamma_i(m,m'), \\
\beta_{i-1}(m) &= \sum_{m'} \beta_i(m')\gamma_i(m,m').
\end{aligned}
$$

We assume that the encoder clears the register at the end of the encoding operation for each codeword. Then, the boundary values for $\alpha_i(m')$ and $\beta_i(m)$ are $\alpha_0(0) = 1$ and $\alpha_0(m) = 0$ for $m \neq 0$, $\beta_l(0) = 1$ and $\beta_l(m) = 0$ for $m \neq 0$.

B. Calculation of Parity Symbol Soft Estimates for the Interleaved Source Information

In the next step, the relay uses the APPs of the information symbols, $P(b_k = w \,|\, \mathbf{y}_{\mathrm{sr}}), k = 1, \ldots, l$, to calculate the parity symbol soft estimates for the interleaved information symbols.

Let

$$
\tilde{\mathbf{B}} = (\tilde{b}_1, \ldots, \tilde{b}_k, \ldots, \tilde{b}_l) = (b_{p_1}, \ldots, b_{p_k}, \ldots, b_{p_l}) \qquad (9)
$$

represent the interleaved version of the binary information stream $\mathbf{B}$, where $\tilde{b}_k, k = 1, \ldots, l$ is the kth symbol of $\tilde{\mathbf{B}}$, and $\tilde{b}_k = b_{p_k}$ represents the interleaving operation in which the p_kth bit in $\mathbf{B}$ becomes the kth symbol in $\tilde{\mathbf{B}}$.

Let $\tilde{\mathbf{C}}$ be the vector of parity symbols of $\tilde{\mathbf{B}}$, denoted by

$$
\tilde{\mathbf{C}} = (\tilde{c}_1, \ldots, \tilde{c}_k, \ldots, \tilde{c}_l) \qquad (10)
$$

where $\tilde{c}_k$ is the parity symbol of $\tilde{b}_k$.

Let

$$
\mathbf{P_B} = \{P(b_k = w \,|\, \mathbf{y}_{\mathrm{sr}}), w = 0, 1, k = 1, \ldots, l\} \qquad (11)
$$

represent the set of the APPs of the information symbols.

Let

$$
\mathbf{P_{\tilde{B}}} = \{P(\tilde{b}_k = w \,|\, \mathbf{y}_{\mathrm{sr}}), w = 0, 1, k = 1, \ldots, l\} \qquad (12)
$$

where $P(\tilde{b}_k = w \,|\, \mathbf{y}_{\mathrm{sr}}) = P(b_{p_k} = w \,|\, \mathbf{y}_{\mathrm{sr}})$ is the APP of the kth interleaved information symbol, which is also the APP of the p_kth uninterleaved information symbol.

Now, let us calculate the APP of $\tilde{c}_k$ given $\mathbf{P_B}$, or equivalently, given $\mathbf{P_{\tilde{B}}}$, denoted by

$$
P(\tilde{c}_k = \tilde{w}_c \,|\, \mathbf{y}_{\mathrm{sr}}, \mathbf{P_{\tilde{B}}}), w_c \in \{0, 1\}. \qquad (13)
$$

We calculate this probability in the following recursive format:

$$
\begin{aligned}
&P(\tilde{c}_k = \tilde{w}_c \,|\, \mathbf{y}_{\mathrm{sr}}, \mathbf{P_{\tilde{B}}}) \\
&= \sum_{m \in U(\tilde{c}_k = \tilde{w}_c)} P(\tilde{b}_k = \tilde{w}_b \,|\, \mathbf{y}_{\mathrm{sr}}, \mathbf{P_{\tilde{B}}}, g(k-1) = m) \\
&\qquad \times P(g(k-1) = m \,|\, \mathbf{y}_{\mathrm{sr}}, \mathbf{P_{\tilde{B}}}) \\
&= \sum_{m \in U(\tilde{c}_k = \tilde{w}_c)} P(\tilde{b}_k = \tilde{w}_b \,|\, \mathbf{y}_{\mathrm{sr}}) \\
&\qquad \times P(g(k-1) = m \,|\, \mathbf{y}_{\mathrm{sr}}, \mathbf{P_{\tilde{B}}}) \qquad (14)
\end{aligned}
$$

$$
\begin{aligned}
&P(g(k) = m \,|\, \mathbf{y}_{\mathrm{sr}}, \mathbf{P_{\tilde{B}}}) \\
&= \sum_{m'} P(g(k) = m | g(k-1) = m', \mathbf{y}_{\mathrm{sr}}, \mathbf{P_{\tilde{B}}}) \\
&\qquad \times P(g(k-1) = m' \,|\, \mathbf{y}_{\mathrm{sr}}, \mathbf{P_{\tilde{B}}}) \\
&= \sum_{m'} P(b(m, m') \,|\, \mathbf{y}_{\mathrm{sr}}) \\
&\qquad \times P(g(k-1) = m' | \mathbf{y}_{\mathrm{sr}}, \mathbf{P_{\tilde{B}}}). \qquad (15)
\end{aligned}
$$

where $U(\tilde{c}_k = \tilde{w}_c)$ is the set of branches, for which the output parity symbol is equal to $\tilde{w}_c$, $\tilde{w}_b$ is the output information symbol corresponding to the parity symbol $\tilde{w}_c$ and the trellis state $g(k-1) = m$. $P(\tilde{b}_k = \tilde{w}_b \,|\, \mathbf{y}_{\mathrm{sr}}, \mathbf{P_{\tilde{B}}}, g(k-1) = m)$ represents the APP of information symbol $\tilde{w}_b$ at time k, which is equivalent to $P(\tilde{b}_k = \tilde{w}_b \,|\, \mathbf{y}_{\mathrm{sr}})$. $P(g(k-1) = m \,|\, \mathbf{y}_{\mathrm{sr}}, \mathbf{P_{\tilde{B}}})$ is the probability of the state m at time $k-1$, $b(m, m')$ represents the input information symbol resulting in the transition from state m' at time $(k-1)$ to m at time k and $P(b(m, m') \,|\, \mathbf{y}_{\mathrm{sr}})$ is the APP of information symbol $b(m, m')$, at time k.

We assume that the encoder clears the register at the end of the encoding operation for each codeword, resulting in the initial state probabilities

$$
P(g(0) = 0) = 1 \text{ and } P(g(0) = m, m \neq 0) = 0. \qquad (16)
$$

From (14) and (15), we can recursively calculate the APPs of $\tilde{\mathbf{C}}$ by using the APPs of $\mathbf{B}$. In (14)–(16), we only consider a forward recursive calculation of the APP of $\tilde{\mathbf{C}}$. A more exact calculation method can be derived by using both forward and backward recursive operations, in which more calculations are needed.

We assume that in the considered BPSK modulation signal set, the binary symbols 0 and 1 are mapped into 1 and -1, respectively. The parity symbol soft estimates of $\tilde{c}_k, k = 1, \ldots, l$, denoted by $\hat{s}_k^p$, can then be calculated as follows:

$$\hat{s}_k^p = P(\tilde{c}_k = 0 \,|\, \mathbf{y}_{\mathbf{sr}}, \mathbf{P}_{\hat{\mathbf{B}}}) \cdot 1 \\ + P(\tilde{c}_k = 1 \,|\, \mathbf{y}_{\mathbf{sr}}, \mathbf{P}_{\hat{\mathbf{B}}}) \cdot (-1). \quad (17)$$

We can write $\hat{s}_k^p$ as $\hat{s}_k^p = \tilde{s}_k^p + \bar{n}_k$, where $\tilde{s}_k^p$ is the exact parity symbol of $\tilde{b}_k$ and $\bar{n}_k = \hat{s}_k^p - \tilde{s}_k^p$ is an equivalent noise, which can be modeled as a Gaussian random variable with a zero-mean and variance $\sigma_{\bar{n}}^2$.

If $\tilde{s}_k^p$ and $\bar{n}_k$ are independent, then the average power of $\hat{s}_k^p$ is

$$E(|\hat{s}_k^p|^2) = E(|\tilde{s}_k^p|^2 + |\bar{n}_k|^2) = 1 + \sigma_{\bar{n}}^2 > 1. \quad (18)$$

However, we can note from (17) that $-1 \leq \hat{s}_k^p \leq 1$ and $|\hat{s}_k^p|^2 \leq 1$, which contradicts (18). This means that $\tilde{s}_k^p$ and $\bar{n}_k$ are not independent. We introduce another equivalent model of $\hat{s}_k^p$ and write $\hat{s}_k^p$ in the following manner:

$$\hat{s}_k^p = \tilde{s}_k^p(1 - \bar{n}_k) \quad (19)$$

where $\bar{n}_k \geq 0$ is an equivalent noise, with the mean

$$\mu_{\bar{n}} = \frac{1}{l} \sum_{k=1}^{l} \bar{n}_k = \frac{1}{l} \sum_{k=1}^{l} (1 - \hat{s}_k^p \tilde{s}_k^p) \\ = \frac{1}{l} \sum_{k=1}^{l} |\hat{s}_k^p - \tilde{s}_k^p| \quad (20)$$

and variance

$$\sigma_{\bar{n}}^2 = \frac{1}{l} \sum_{k=1}^{l} (1 - \hat{s}_k^p \tilde{s}_k^p - \mu_{\bar{n}}). \quad (21)$$

The signals transmitted from the relay can then be written as

$$x_{r,k} = \beta \hat{s}_k^p = \beta \tilde{s}_k^p(1 - \bar{n}_k) \quad (22)$$

where β is a normalization factor calculated from the transmitted power constraint at the relay

$$E(|x_{r,k}|^2) = \beta^2 \left(1 - 2\mu_{\bar{n}} + E\left(\bar{n}_k^2\right)\right) \\ = \beta^2 \left((1 - \mu_{\bar{n}})^2 + \sigma_{\bar{n}}^2\right) \leq P_2 \quad (23)$$

where P_2 is the transmitted power limit at the relay.

Let $\bar{\omega} = (1 - \mu_{\bar{n}})^2 + \sigma_{\bar{n}}^2$, then from (23), we can obtain

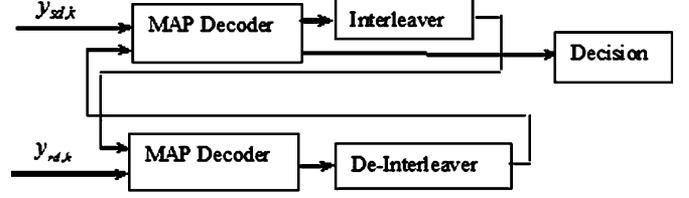$$\beta \leq \sqrt{\frac{P_2}{\bar{\omega}}}. \quad (24)$$

Fig. 4. Block diagram of decoder.

Since the source and the relay are transmitted in different time slots, the destination receiver can separate them. The destination received signal, corresponding to the relay signal at time k, denoted by $y_{\mathrm{rd},k}$, can be written as

$$y_{\mathrm{rd},k} = G_{\mathrm{rd}} h_{\mathrm{rd}} x_{r,k} + n_{\mathrm{rd},k} \\ = G_{\mathrm{rd}} h_{\mathrm{rd}} \beta \tilde{s}_k^p(1 - \mu_{\tilde{n}}) + \bar{n}_{\mathrm{rd},k} \quad (25)$$

where $\bar{n}_{\mathrm{rd},k} = n_{\mathrm{rd},k} - G_{\mathrm{rd}} h_{\mathrm{rd}} \beta \tilde{s}_k^p(\bar{n}_k - \mu_{\bar{n}})$ is the equivalent noise at the destination for DTC-SIR, with a zero-mean and variance of σ_E^2, given by

$$\sigma_E^2 = N_0 + G_{\mathrm{rd}}^2 |h_{\mathrm{rd}} \beta|^2 \sigma_{\bar{n}}^2. \quad (26)$$

The destination received signal, corresponding to the signal transmitted from the source is given in (5). From (5) and (25), we can observe that an overall distributed codeword consists of the coded information symbols transmitted from the source, given in (5), and the parity symbols of the interleaved information sequence sent from the relay, shown in (25). These two signals at the destination are denoted by $\mathbf{y}_{\mathrm{sd},k}$ and $y_{\mathrm{rd},k}$, respectively. They are decoded by two separate decoders.

A turbo iterative decoding algorithm is performed between these two decoders. Fig. 4 shows a block diagram of the iterative decoder. Iterations are performed between the two decoders associated with $\mathbf{y}_{\mathrm{sd},k}$ and $y_{\mathrm{rd},k}$. The MAP decoders with input symbols $\mathbf{y}_{\mathrm{sd},k}$ and $y_{\mathrm{rd},k}$ calculate the APP and extrinsic information of the transmitted information symbols and the interleaved information symbols, respectively. The extrinsic information of one decoder is used to update the *a priori* probability of the other decoder in the next iteration. After several iterations, the decision is made based on the APPs of the first decoder.

We should point out that the calculation of variable $\gamma_k(m, m')$ in the decoder, associated with $y_{\mathrm{rd},k}$, is somewhat different from (8) and is given by

$$\gamma_k(m, m') = \exp\left(-\frac{|y_{\mathrm{rd},k} - G_{\mathrm{rd}} h_{\mathrm{rd}} \beta \tilde{s}_k^p(1 - \mu_{\bar{n}})|^2}{\sigma_E^2}\right).$$

IV. PERFORMANCE ANALYSIS OF THE DTC-SIR

In this section, we analyze the performance of the DTC-SIR. The calculation of the traditional union bound requires knowing the number of codewords with various Hamming weights, which needs an exhaustive search of the code trellis. Due to a

high complexity of this search, we consider an average upper bound [20].

Let us first calculate the average destination received SNR for the DTC-SIR and the perfect DTC, respectively.

Let $\gamma_{r,\text{in}}$ and $\gamma_{r,\text{out}}$ be the input and output SNR at the relay decoder, respectively, where the input and output signals at the relay decoder are the received signals at the relay and the parity symbol soft estimate $\hat{s}_k^p$, given in (17), respectively. The relationship between $\gamma_{r,\text{in}}$ and $\gamma_{r,\text{out}}$ can be represented by a function $f(x)$

$$\gamma_{r,\text{out}} = f(\gamma_{r,\text{in}}) \tag{27}$$

where $\gamma_{r,\text{in}} = (P_{\text{sr}}|h_{\text{sr}}|^2)/N_0$ and $\gamma_{r,\text{out}} = (1 - \mu_{\bar{n}})^2/\sigma_{\bar{n}}^2$, which can be calculated from (19)–(21).

For a convolutional code, the asymptotic soft decision coding gain at a high SNR can be approximated by

$$\gamma_c = 10\log(Rd_{\text{free}})\text{dB} \tag{28}$$

where R is the code rate and d_{free} is the code minimum free distance. This gain goes down with reducing the SNR. Therefore, for convolutional codes, the decoder SNR input/output relationship can be approximated as

$$\gamma_{r,\text{out}} \leq \gamma_{r,\text{in}}Rd_{\text{free}}. \tag{29}$$

The destination received SNR, corresponding to the signals transmitted from the relay, denoted by γ_{RD}, can be calculated from (25)–(29) as

$$
\begin{aligned}
\gamma_{RD} &= \frac{G_{\text{rd}}^2\beta^2|h_{\text{rd}}|^2(1-\mu_{\bar{n}})^2}{G_{\text{rd}}^2\beta^2|h_{\text{rd}}|^2\sigma_{\bar{n}}^2 + N_0} \\
&\overset{(a)}{\leq} \frac{\frac{P_{\text{rd}}/N_0}{(1-\mu_{\bar{n}})^2+\sigma_{\bar{n}}^2}|h_{\text{rd}}|^2(1-\mu_{\bar{n}})^2}{\frac{P_{\text{rd}}/N_0}{(1-\mu_{\bar{n}})^2+\sigma_{\bar{n}}^2}|h_{\text{rd}}|^2\sigma_{\bar{n}}^2 + 1} \\
&= \frac{\frac{P_{\text{rd}}}{N_0}|h_{\text{rd}}|^2\frac{(1-\mu_{\bar{n}})^2/\sigma_{\bar{n}}^2}{1+(1-\mu_{\bar{n}})^2/\sigma_{\bar{n}}^2}}{\frac{P_{\text{rd}}/N_0}{1+(1-\mu_{\bar{n}})^2/\sigma_{\bar{n}}^2}|h_{\text{rd}}|^2 + 1} \\
&\overset{(b)}{\leq} \frac{|h_{\text{rd}}|^2|h_{\text{sr}}|^2 Rd_{\text{free}}\gamma_{s-r}\gamma_{r-d}}{|h_{\text{rd}}|^2\gamma_{r-d} + |h_{\text{sr}}|^2 Rd_{\text{free}}\gamma_{s-r} + 1} \\
&\leq \left(\frac{1}{|h_{\text{sr}}|^2 Rd_{\text{free}}\gamma_{s-r}} + \frac{1}{|h_{\text{rd}}|^2\gamma_{r-d}}\right)^{-1} \\
&\overset{(c)}{=} \frac{1}{2}\left(\frac{1}{2}\sum_{p=1}^2 \frac{1}{\lambda_p}\right)^{-1} \tag{30}
\end{aligned}
$$

where
a) follows from $\beta \leq \sqrt{P_2/((1-\mu_{\bar{n}})^2 + \sigma_{\bar{n}}^2)}$ and $P_{\text{rd}} = P_2 G_{\text{rd}}^2$ is the received power at the destination for the signals transmitted from the relay;
b) from $\gamma_{r,\text{out}} = (1-\mu_{\bar{n}})^2/\sigma_{\bar{n}}^2 \leq \gamma_{r,\text{in}}Rd_{\text{free}}, \gamma_{r,\text{in}} = (P_{\text{sr}}|h_{\text{sr}}|^2)/N_0, \gamma_{s-r} = P_{\text{sr}}/N_0, \gamma_{r-d} = P_{\text{rd}}/N_0$;
c) $\lambda_1 = |h_{\text{sr}}|^2 Rd_{\text{free}}\gamma_{s-r}, \lambda_2 = |h_{\text{rd}}|^2\gamma_{r-d}$, and $((1/2)\sum_{p=1}^2(1/\lambda_p))^{-1}$ is called the Harmonic Mean

[2], [5], [15] of variables $\lambda_p, p = 1, 2$, and is denoted by $H_2^{\lambda_p}$.

The average γ_{RD}, denoted by $\bar{\gamma}_{RD}$, can be obtained by averaging (30) with respect to $|h_{\text{sr}}|^2$ and $|h_{\text{rd}}|^2$.

Let $\varphi_{\text{sr}} = |h_{\text{sr}}|^2$ and $\varphi_{\text{rd}} = |h_{\text{rd}}|^2$. The probability density function (pdf) of φ_{sr} and φ_{rd}, denoted by $p(\varphi_{\text{sr}})$ and $p(\varphi_{\text{rd}})$, are given by $p(\varphi_{\text{sr}}) = e^{-\varphi_{\text{sr}}}$ and $p(\varphi_{\text{rd}}) = e^{-\varphi_{\text{rd}}}$, respectively. The pdf of $H_2^{\lambda_p}$ is given by [26]

$$
\begin{aligned}
&p\left(H_2^{\lambda_p}\right) \\
&= \frac{2H_2^{\lambda_p}\exp\left(-H_2^{\lambda_p}\left(\frac{1}{Rd_{\text{free}}\gamma_{s-r}} + \frac{1}{\gamma_{r-d}}\right)\right)}{Rd_{\text{free}}\gamma_{s-r}\gamma_{r-d}} \\
&\quad \times \left[\frac{Rd_{\text{free}}\gamma_{s-r} + \gamma_{r-d}}{\sqrt{Rd_{\text{free}}\gamma_{s-r}\gamma_{r-d}}}K_1\left(\frac{2H_2^{\lambda_p}}{\sqrt{Rd_{\text{free}}\gamma_{s-r}\gamma_{r-d}}}\right)\right. \\
&\quad \left.+ 2K_0\left(\frac{2H_2^{\lambda_p}}{\sqrt{Rd_{\text{free}}\gamma_{s-r}\gamma_{r-d}}}\right)\right] \tag{31}
\end{aligned}
$$

where $K_0(x)$ and $K_1(x)$ are the zeroth and first-order modified Bessel function of the second kind, respectively.

The average γ_{RD} can be calculated as [26]

$$
\begin{aligned}
&\bar{\gamma}_{RD} \\
&= \frac{1}{2}\int_0^\infty H_2^{\lambda_p}p\left(H_2^{\lambda_p}\right)dH_2^{\lambda_p} \\
&= \frac{32}{15}\left(\frac{1}{\sqrt{Rd_{\text{free}}\gamma_{s-r}}} + \frac{1}{\sqrt{\gamma_{r-d}}}\right)^{-6} \\
&\quad \times F\left(3, \frac{1}{2}; 3\frac{1}{2}; \left(\frac{\sqrt{Rd_{\text{free}}\gamma_{s-r}} - \sqrt{\gamma_{r-d}}}{\sqrt{Rd_{\text{free}}\gamma_{s-r}} + \sqrt{\gamma_{r-d}}}\right)^2\right) \\
&\quad + \frac{32}{5}\frac{Rd_{\text{free}}\gamma_{s-r} + \gamma_{r-d}}{Rd_{\text{free}}\gamma_{s-r}\gamma_{r-d}} \\
&\quad \times \left(\frac{1}{\sqrt{Rd_{\text{free}}\gamma_{s-r}}} + \frac{1}{\sqrt{\gamma_{r-d}}}\right)^{-8} \\
&\quad \times F\left(4, \frac{3}{2}; 3\frac{1}{2}; \left(\frac{\sqrt{Rd_{\text{free}}\gamma_{s-r}} - \sqrt{\gamma_{r-d}}}{\sqrt{Rd_{\text{free}}\gamma_{s-r}} + \sqrt{\gamma_{r-d}}}\right)^2\right) \tag{32}
\end{aligned}
$$

where $F(a, b; c; d)$ is a hypergeometric function.

For perfect DTC, the SNR at the destination, corresponding to the signal transmitted from the relay, denoted by $\gamma_{RD}^{\text{Perfect}}$, is given by

$$\gamma_{RD}^{\text{Perfect}} = \frac{P_{\text{rd}}|h_{\text{rd}}|^2}{N_0} = \gamma_{r-d}|h_{\text{rd}}|^2. \tag{33}$$

Its average value, denoted by $\bar{\gamma}_{RD}^{\text{Perfect}}$, can be calculated as

$$\bar{\gamma}_{RD}^{\text{Perfect}} = E\left(\frac{P_{\text{rd}}|h_{\text{rd}}|^2}{N_0}\right) = \gamma_{r-d}. \tag{34}$$

Now, let us calculate the average error probability upper bound of the DTC-SIR.

As stated in Section III, a codeword of the DTC system, consists of the coded information symbols $\mathbf{S}$, transmitted from the source and the parity of the interleaved information symbols, denoted by $\mathbf{S}_r$, sent from the relay. These two signals have different SNRs at the destination, denoted by γ_{SD} and γ_{RD}, respectively.

Assume that an all-zero codeword is transmitted, then the pairwise error probability (PEP) that the decoder decides in favor of another erroneous codeword with Hamming weight d, for linear codes, is given by [13], [19]

$$P(d|h_{\mathrm{sd}}, h_{\mathrm{rd}}, h_{\mathrm{sr}}) = Q(\sqrt{2d_1\gamma_{SD} + 2d_2\gamma_{RD}}) \quad (35)$$

where γ_{RD} is given by (30), $\gamma_{SD} = (P_{\mathrm{sd}}/N_0)|h_{\mathrm{sd}}|^2 = \gamma_{s-d}|h_{\mathrm{sd}}|^2$, $\gamma_{s-d} = P_{\mathrm{sd}}/N_0$, d_1 and d_2 are the Hamming weights of the erroneous codewords with Hamming weight d, transmitted from the source and the relay, respectively, such that $d = d_1 + d_2$.

The average sequence error probability can be derived by averaging (35) over the fading coefficients. Let $P(d)$ be the average probability of decoding an erroneous code sequence with weight d, then

$$\begin{aligned} p(d) &= \int_{h_{\mathrm{rd}}} \int_{h_{\mathrm{sr}}} \int_{h_{\mathrm{sd}}} P(d \,|\, h_{\mathrm{sd}}, h_{\mathrm{rd}}, h_{\mathrm{sr}}) \\ &\quad \times p(h_{\mathrm{sd}})p(h_{\mathrm{sr}})p(h_{\mathrm{rd}})d(h_{\mathrm{sd}})d(h_{\mathrm{sr}})d(h_{\mathrm{rd}}) \\ &\leq \frac{1}{2}(d_1\gamma_{s-d})^{-1}f(d_2) \end{aligned} \quad (36)$$

where $p(h_{\mathrm{sd}}), p(h_{\mathrm{sr}})$, and $p(h_{\mathrm{rd}})$ are the pdf of $h_{\mathrm{sd}}, h_{\mathrm{sr}}$ and h_{rd}, respectively, and

$$\begin{aligned} f(d_2) &= \int_{h_{\mathrm{rd}}} \int_{h_{\mathrm{sr}}} Q(\sqrt{2d_2\gamma_{RD}}) \\ &\quad \times p(h_{\mathrm{rd}})p(h_{\mathrm{sr}})d(h_{\mathrm{rd}})d(h_{\mathrm{sr}}) \\ &= \int Q\left(\sqrt{4d_2 H_2^{\lambda_p}}\right) p(H_2^{\lambda_p})dH_2^{\lambda_p} \end{aligned} \quad (37)$$

where $p(H_2^{\lambda_p})$ is given in (31).

The closed-form expression of $f(d_2)$ can be calculated by using the moment generating function (MDF) of the Harmonic mean of two exponential random variables [5], [26]. The exact closed form expression of $f(d_2)$ is too complex to be presented here. At a high SNR, $f(d_2)$ can be approximated as [14], [18]

$$f(d_2) \simeq \left(\frac{1}{Rd_{\mathrm{free}}\gamma_{s-r}} + \frac{1}{\gamma_{r-d}}\right)(d_2)^{-1}. \quad (38)$$

Let P_b be the average upper bound on the bit-error rate (BER). At high SNR, P_b can be approximated as

$$\begin{aligned} P_b &\leq \sum_{d=d_{\min}}^{3l} \sum_{i=1}^{l} \frac{i}{l} \binom{l}{i} \bar{A}(d)P(d) \\ &\simeq \frac{1}{2}(\gamma_{s-d})^{-1} \left(\frac{1}{Rd_{\mathrm{free}}\gamma_{s-r}} + \frac{1}{\gamma_{r-d}}\right) \\ &\quad \times \sum_{d=d_{\min}}^{3l} \sum_{i=1}^{l} \frac{i}{l} \binom{l}{i} \bar{A}(d)(d_1 d_2)^{-1} \end{aligned} \quad (39)$$

where $\bar{A}(d) = \sum_{i=1}^{l} \binom{l}{i} p(d\,|\,i)$, $\binom{l}{i}$ is the number of codewords with Hamming weight i and $p(d\,|\,i)$ is the probability that an input codeword with Hamming weight i produces a codeword with Hamming weight d.

Similarly, for the perfect DTC, the average PEP of incorrectly decoding to a codeword with weight d, denoted by $P^{\mathrm{perfect}}(d)$, can be calculated as

$$P^{\mathrm{perfect}}(d) \leq \frac{1}{2}(d_1\gamma_{s-d})^{-1}(d_2\gamma_{r-d})^{-1}. \quad (40)$$

Let P_b^{Perfect} be the average upper bound on the BER of perfect DTC. It can be expressed as

$$\begin{aligned} P_b^{\mathrm{Perfect}} &\leq \sum_{d=d_{\min}}^{3l} \sum_{i=1}^{l} \frac{i}{l} \binom{l}{i} \bar{A}(d) \\ &\quad \times P^{\mathrm{Perfect}}(d) \\ &\simeq \frac{1}{2}(\gamma_{s-d})^{-1}(\gamma_{r-d})^{-1} \\ &\quad \times \sum_{d=d_{\min}}^{3l} \sum_{i=1}^{l} \frac{i}{l} \binom{l}{i} \bar{A}(d)(d_1 d_2)^{-1}. \end{aligned} \quad (41)$$

Due to imperfect decoding at the relay, the DTC-SIR has a performance loss compared with the perfect DTC. We define $\mathrm{SNR}_{\mathrm{loss}}$ as the difference in SNR to achieve the same BER between the DTC-SIR and the perfect DTC. Comparing (39) and (41), we can see that the SNR loss of the DTC-SIR compared with the perfect DTC, denoted by $\mathrm{SNR}_{\mathrm{loss}}$, can be approximated as

$$\mathrm{SNR}_{\mathrm{loss}} \simeq \frac{\gamma_{r-d}}{Rd_{\mathrm{free}}\gamma_{s-r}} + 1 = \frac{1}{Rd_{\mathrm{free}}\gamma_{\mathrm{Gap}}} + 1 \quad (42)$$

where $\gamma_{\mathrm{Gap}} = \gamma_{s-r}/\gamma_{r-d}$ and

$$\mathrm{SNR}_{\mathrm{Gap}}(\mathrm{dB}) = 10\log(\gamma_{\mathrm{Gap}}) = 10\log(\gamma_{s-r}/\gamma_{r-d}) \quad (43)$$

represent the difference in SNR between γ_{s-r} and γ_{r-d} in decibels.

From (42), we can observe that in order to decrease the gap between the DTC-SIR and the perfect DTC, we should make Rd_{free} as large as possible and the $\mathrm{SNR}_{\mathrm{Gap}}$ between γ_{s-r} and γ_{r-d} as large as possible.

We should point out that the union bound is tight for independent fast fading channels only [17]–[19]. In order to get a tight upper bound over quasi-static fading channels, we should limit the conditional upper bound in (36), (39), and (41) before averaging over the fading coefficients [19], but no closed-form expression can be obtained, as shown in [19]. Since we are only interested in the relative performance loss of the DTC-SIR compared with the perfect DTC, shown in (42), the average union bound is sufficient for evaluating this performance loss. The calculation of the exact upper bound using the "limit before averaging" method from [19] is out of the scope of this paper.

V. OUTAGE PROBABILITY BOUND OF SOFT INFORMATION RELAYING

In this section, we calculate the outage probability bound of a soft information relaying (SIR). It is obvious that the outage probability is lower bounded by the perfect relay (PR)[2] and upper bounded by the amplify and forward relaying protocol (AAF).[3]

Let $P_{\mathrm{SIR}}^{\mathrm{out}}, P_{\mathrm{PR}}^{\mathrm{out}}$, and $P_{\mathrm{AAF}}^{\mathrm{out}}$ denote the outage probability of SIR, PR, and AAF, respectively, then we have

$$P_{\mathrm{PR}}^{\mathrm{out}} < P_{\mathrm{SIR}}^{\mathrm{out}} < P_{\mathrm{AAF}}^{\mathrm{out}}. \tag{44}$$

For the AAF, the maximum average mutual information in bit/s/Hertz, denoted by I_{AF}, is given by [24]

$$I_{\mathrm{AF}} = \frac{T_s}{T_s + T_r} \log \left(1 + \gamma_{s-d}|h_{\mathrm{sd}}|^2 + \gamma_{s-r-d}^{\mathrm{AAF}}\right) \tag{45}$$

where the coefficient $(T_s)/(T_s + T_r)$ is due to the assumption of a time division duplex format, T_s and T_r represent the transmission duration from the source and the relay, and

$$\gamma_{s-r-d}^{\mathrm{AAF}} = \frac{\gamma_{s-r}\gamma_{r-d}|h_{\mathrm{sr}}|^2|h_{\mathrm{rd}}|^2}{\gamma_{s-r}|h_{\mathrm{sr}}|^2 + \gamma_{r-d}|h_{\mathrm{rd}}|^2 + 1}. \tag{46}$$

For simplicity, we assume that

$$\gamma_{s-d} = \gamma_{r-d} = \gamma \text{ and } \gamma_{s-r} = \gamma_{\mathrm{Gap}}\gamma \tag{47}$$

where $\gamma_{\mathrm{Gap}} = \gamma_{s-r}/\gamma_{r-d}$

Substituting (47) into (45), we get at high SNR

$$I_{\mathrm{AF}} = \frac{T_s}{T_s + T_r} \log \Bigg(1 + \gamma|h_{\mathrm{sd}}|^2 \\ + \frac{\gamma_{\mathrm{Gap}}\gamma^2|h_{\mathrm{sr}}|^2|h_{\mathrm{rd}}|^2}{\gamma_{\mathrm{Gap}}\gamma|h_{\mathrm{sr}}|^2 + \gamma|h_{\mathrm{rd}}|^2 + 1}\Bigg). \tag{48}$$

From (48), the outage event for a spectral efficiency η is determined by $I_{\mathrm{AF}} \leq \eta$. The outage probability for Rayleigh-fading channels, at high SNR, can be approximated as [24]

$$P_{\mathrm{AAF}}^{\mathrm{out}}(\gamma, \eta) := P(I_{\mathrm{AF}} < \eta) \overset{\gamma \to \infty}{\to} \\ \left(\frac{1 + \gamma_{\mathrm{Gap}}}{2\gamma_{\mathrm{Gap}}}\right)\left(\frac{2^{\eta(1+T_r/T_s)} - 1}{\gamma}\right)^2. \tag{49}$$

For the perfect relay, $\gamma_{\mathrm{Gap}} \to \infty$, and (49) is hence approximated to

$$P_{\mathrm{PR}}^{\mathrm{out}}(\gamma, \eta) := \frac{1}{2}\left(\frac{2^{\eta(1+T_r/T_s)} - 1}{\gamma}\right)^2. \tag{50}$$

[2]PR is a relay scheme for which relay can always perform an error-free decoding

[3]AAF is a relay scheme, for which the relay amplifies the received signals and forwards it to the destination

From (49) and (50), we can get the lower and upper bound of the SIR as follows:

$$P_{\mathrm{SIR}}^{\mathrm{out}}(\gamma, \eta) < P_{\mathrm{AAF}}^{\mathrm{out}}(\gamma, \eta) \\ = \left(\frac{1 + \gamma_{\mathrm{Gap}}}{2\gamma_{\mathrm{Gap}}}\right)\left(\frac{2^{\eta(1+T_r/T_s)} - 1}{\gamma}\right)^2 \tag{51}$$

$$P_{\mathrm{SIR}}^{\mathrm{out}}(\gamma, \eta) > P_{\mathrm{PR}}^{\mathrm{out}}(\gamma, \eta) \\ = \frac{1}{2}\left(\frac{2^{\eta(1+T_r/T_s)} - 1}{\gamma}\right)^2. \tag{52}$$

It is clear from the above expressions that the upper bound can approach the lower bound as $\gamma_{\mathrm{Gap}} \gg 1$.

VI. SIMULATION RESULTS AND DISCUSSIONS

In this section, we provide simulation results comparisons for various DTC schemes. All simulations are performed for the BPSK modulation and a frame size of 130 symbols over quasi-static fading channels. We use a four-state RSCC with the code rate of $1/2$ as the turbo component code. The generator matrix of the RSCC is $(1, 5/7)$ with d_{free} of 5. Based on (42), we can calculate the SNR loss of the DTC-SIR, compared with the perfect DTC as

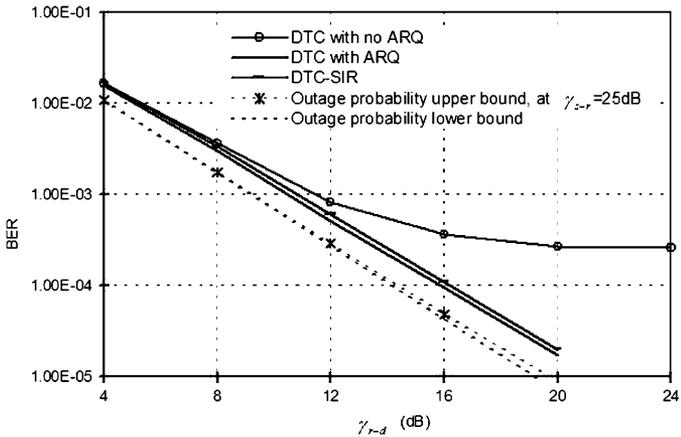$$\mathrm{SNR}_{\mathrm{loss}} = \frac{2\gamma_{r-d}}{5\gamma_{s-r}} + 1 = \frac{2}{5\gamma_{\mathrm{Gap}}} + 1. \tag{53}$$

For simplicity, we also assume that P_{rd} and P_{sd} are the same. Therefore, γ_{s-d} is equal to γ_{r-d}. We carry out the performance comparisons under the following two scenarios.

A. Comparison for Various γ_{s-r}

In this case, we investigate the performance of the DTC schemes when the signal power from the source to the relay is fixed, while the power from the source to the destination and from the relay to the destination are varied.
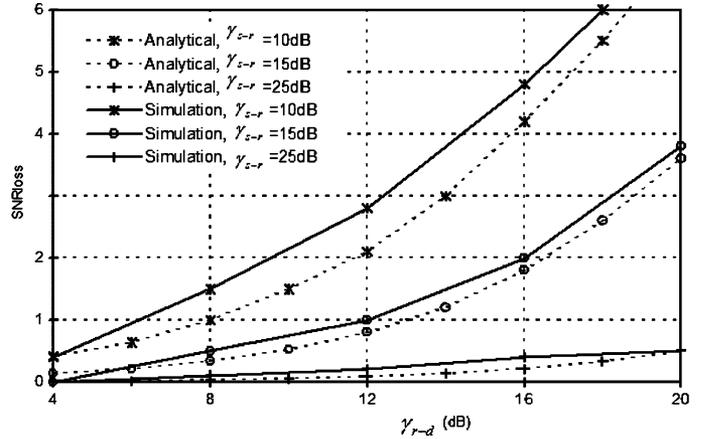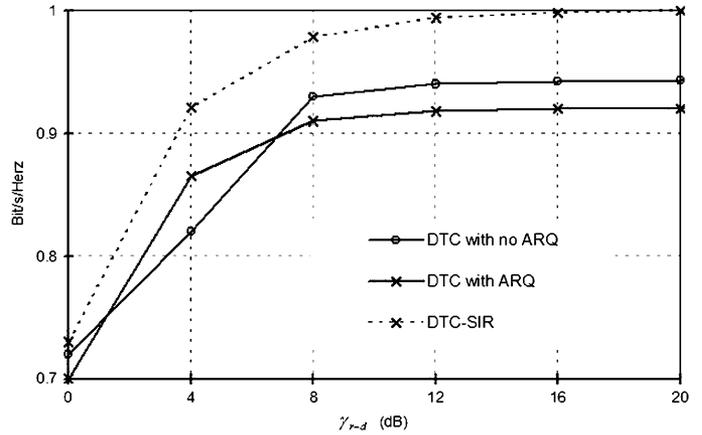
For the DTC with no ARQ, the relay re-encodes the decoded codeword and passes it to the destination regardless whether the relay can make a correct decoding or not. Therefore, when decoding errors occur at the relay, the relay will make an erroneous symbol decision, and re-encoding the erroneous decoded symbols will cause error propagation. We refer to such a scheme as the DTC with no ARQ. In the simulations, we use a Log-Map decoding algorithms at the destination. Unlike the DTC-SIR, in which the equivalent noise variance, corresponding to the destination noise variance and the error variance in soft estimates, can be calculated, as shown in (26), for the DTC with no ARQ, the destination decoder has no knowledge of erroneous codewords introduced by the relay. Thus, the decoder only gets the destination noise variance from the destination in the decoding. This will further degrade its performance.

As stated in Section I, the performance of DTC can be improved by using an ARQ scheme in the link from the source to the relay. Such a scheme performs very close to the perfect DTC. We refer to this scheme as the DTC with ARQ. In the following simulation results, for the DTC with ARQ, we assume
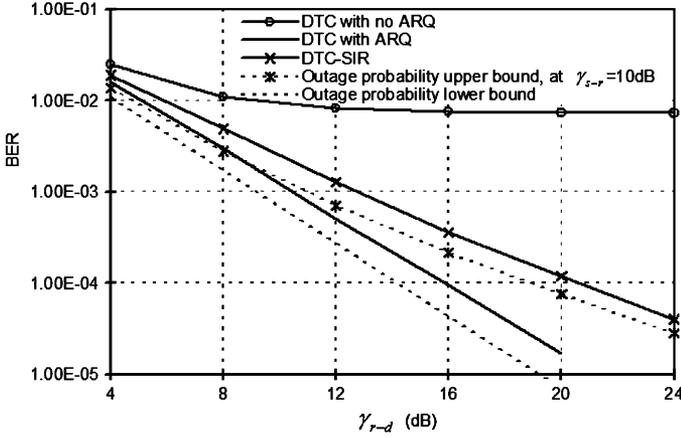
Fig. 5. BER performance comparison at $\gamma_{s-r} = 10$ dB.
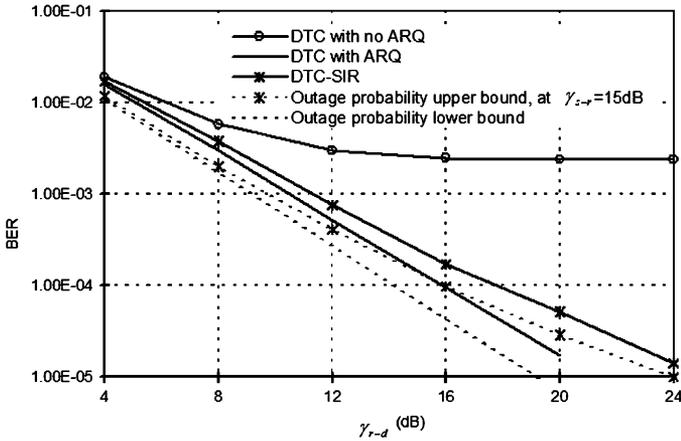
Fig. 6. BER performance comparison at $\gamma_{s-r} = 15$ dB.

Fig. 7. BER performance comparison at $\gamma_{s-r} = 25$ dB.

Fig. 8. SNR loss of the DTC-SIR compared with the perfect DTC for various γ_{s-r}.

Fig. 9. Throughput comparison at $\gamma_{s-r} = 10$ dB.

that the maximum number of retransmissions (MNR) is three and a transmission is said to be a failure if it is still unsuccessful after the maximum number of retransmissions. In this case, the relay will discard the frame.

Figs. 5–7 compare the BER performance of the DTCs with and without ARQ, and the proposed DTC-SIR, as a function of γ_{r-d} and with γ_{s-r} as a parameter. It can be noted from these figures that due to error propagation, the DTC with no ARQ performs badly and achieves an error floor for high γ_{r-d}, especially at lower γ_{s-r}. In contrast, the DTC-SIR can effectively mitigate
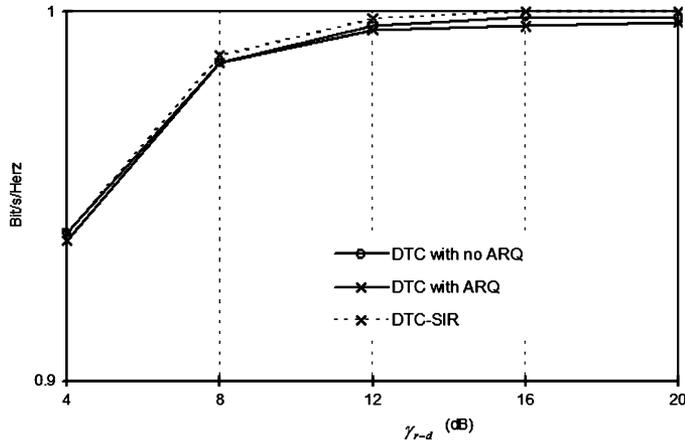
the effect of error propagation. Its performance approaches the DTC with ARQ as γ_{s-r} increases.

Figs. 5–7 also compare the performance of the DTC-SIR with the outage probability lower and upper bounds, described by the perfect relay and the AAF relay system, as discussed in Section V. It can be noted from the figures that the DTC-SIR can approach the upper bound for larger γ_{r-d} within 2 dB for any values of γ_{s-r} and the lower bound within 2 dB at the BER of 10^{-4}, at $\gamma_{s-r} = 25$ dB.

From (42) and (53), we can also observe that if γ_{s-r} is constant and γ_{r-d} varies, then the SNR loss of the DTC-SIR compared with perfect DTC will increase as γ_{r-d} increases. Fig. 8 compares the analytical SNR loss derived from (53) and the one obtained from simulation results in Figs. 5–7.

It can be observed from Fig. 8 that the expression in (53) is tight for large γ_{s-r}. This is because the approximation of the soft decision coding gain γ_c in (28) is tight only for high $\gamma_{r,\text{in}}$ or γ_{s-r}. If we use the exact value of γ_c, the simulation results will be closer to analytical ones. The exact γ_c can be calculated from the exact bit-error probability of the convolutional code [23], but the expression is very complex.

Figs. 9–11 compare the throughput of these DTC schemes. It can be noted that the throughput of the DTC-SIR is higher than both the DTCs with and without ARQ at $\gamma_{s-r} = 10$ dB.
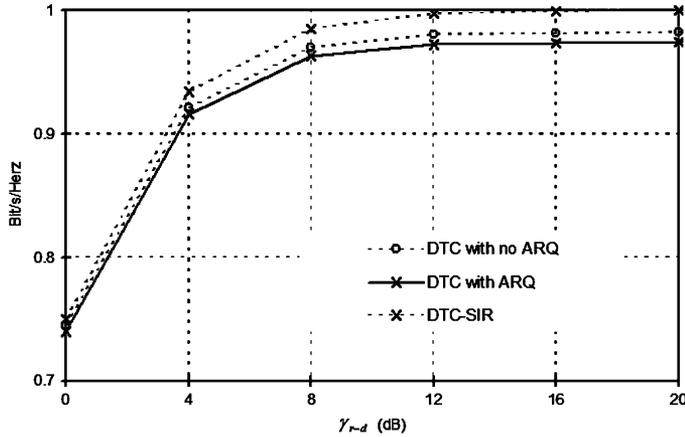
Fig. 10.   Throughput comparison at $\gamma_{s-r} = 15$ dB.

Fig. 12.   BER performance of the proposed scheme at $\mathrm{SNR_{Gap}} = 0$ dB.

Fig. 11.   Throughput comparison at $\gamma_{s-r} = 25$ dB.

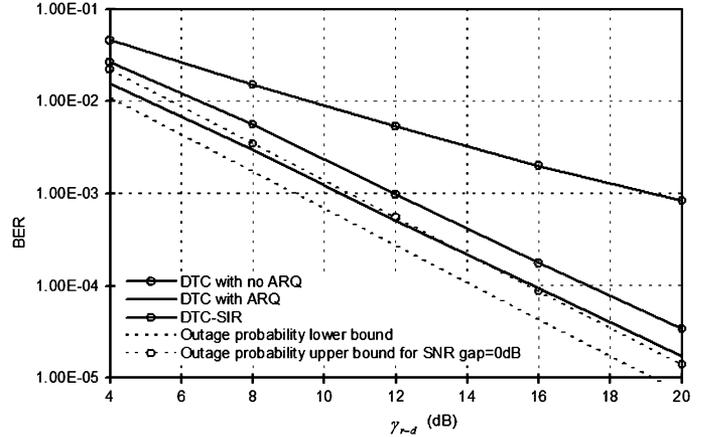Fig. 13.   BER performance of the proposed scheme at $\mathrm{SNR_{Gap}} = 4$ dB.
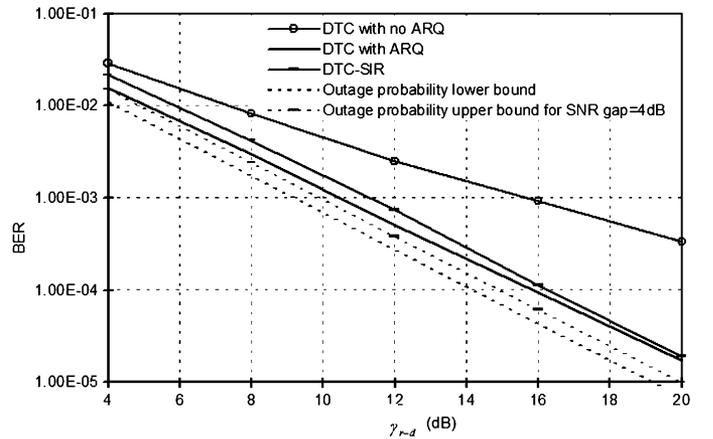
The throughput gap between them decreases as γ_{s-r} increases. Therefore, although the BER performance of the DTC-SIR is worse than that of the DTC with ARQ, the former will significantly improve the throughput of a distributed coding system.

B. Comparison for Various SNR Gaps Between γ_{s-r} and γ_{r-d}

It can be noted from (42) that if we set the $\mathrm{SNR_{Gap}}$ as a constant, $\mathrm{SNR_{loss}}$ will become a constant for all γ_{r-d}. In this case, the BER curve of the DTC-SIR should be parallel to that of the perfect DTC and the gap between them is the $\mathrm{SNR_{loss}}$ given in (53).

We assume that the signal energy decays exponentially with distance between two nodes, as shown in Section II. Obviously, $\mathrm{SNR_{Gap}}$ is determined by the ratio of the power transmitted from the source and the relay, as well as the relative distance of d_{sr} and d_{rd}. More specifically

$$\mathrm{SNR_{Gap}} = 10\log\left(\frac{P_s(G_{\mathrm{sr}})^2}{P_2(G_{\mathrm{rd}})^2}\right)$$
$$= 10\log\left(\frac{P_s}{P_2}\left(\frac{d_{\mathrm{sr}}}{d_{\mathrm{rd}}}\right)^{-\kappa}\right) \qquad (54)$$

where P_s/P_2 and $(d_{\mathrm{sr}}/d_{\mathrm{rd}})^{-\kappa}$ are referred to as the power amplification factor and the geometrical distribution factor, denoted by ρ_P^{sr} and ρ_d^{sr}, respectively.

ρ_P^{sr} denotes the ratio of the source and relay transmit power while ρ_d^{sr} represents the ratio of d_{sr} and d_{rd}, which depends on the network geometrical distribution. In this subsection, we evaluate the performance of the DTC-SIR for various SNR gaps to investigate the effect of the power amplification factor and the network geometrical distribution on the system performance and throughput.

Figs. 12 and 13 show the BER performance comparison for various SNR gaps. It can be observed that the DTC-SIR significantly outperforms the DTC with no ARQ. The performance of the DTC-SIR is worse than that of the DTC with ARQ by about 1.5 and 0.5 dB for $\mathrm{SNR_{Gap}}$ of 0 and 4 dB, at the BER of 10^{-4}, respectively. Therefore, if the $\mathrm{SNR_{Gap}}$ between γ_{s-r} and γ_{r-d} is large enough, then the performance of the DTC-SIR approaches that of the DTC with ARQ. Figs. 12 and 13 also compare the performance of the DTC-SIR with the outage probability lower and upper bound. We can observe that DTC-SIR can approach the outage probability upper and lower bound within 1.5 and 2 dB, at $\mathrm{SNR_{Gap}}$ of 4 dB, at the BER of 10^{-4}.

From (53), we can also calculate that the $\mathrm{SNR_{loss}}$ for $\mathrm{SNR_{Gap}}$ of 0 and 4 dB are 1.46 and 0.64 dB, respectively. Fig. 14 compares the analytical SNR loss derived from (53)
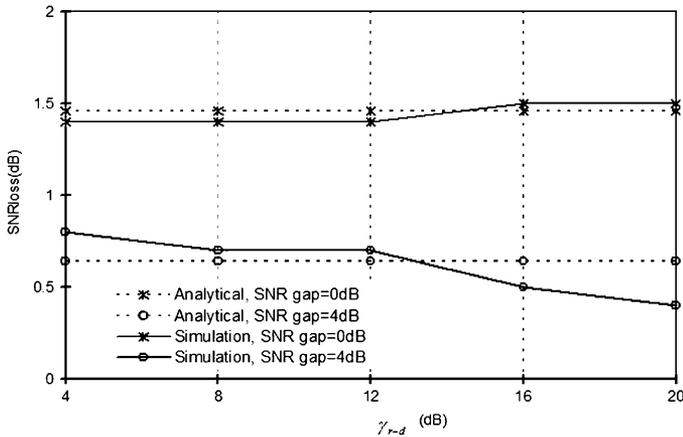
Fig. 14. SNR loss of the DTC-SIR compared with the perfect DTC for various SNR gaps.
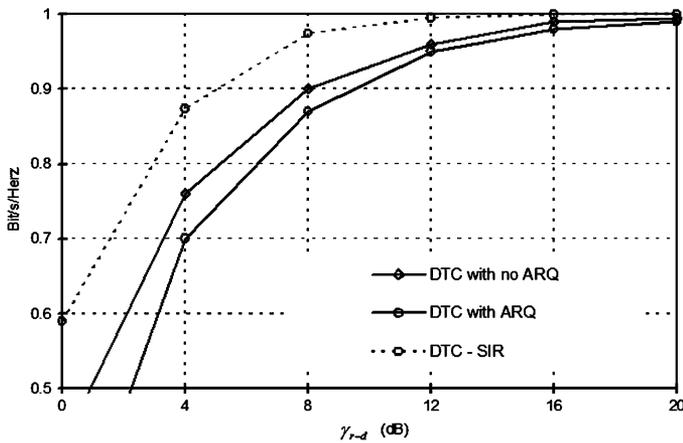

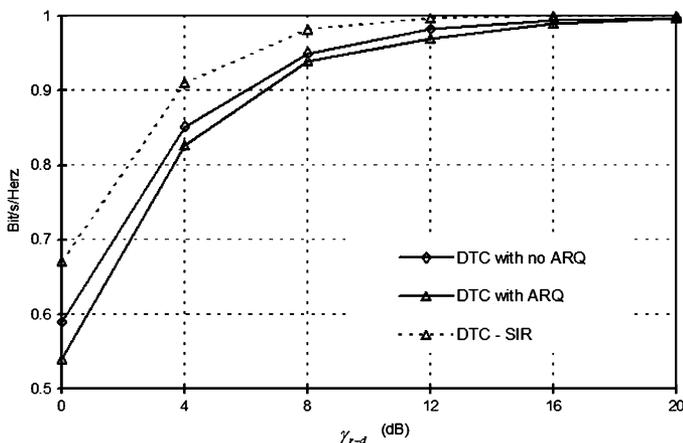Fig. 15. Throughput comparison at $\mathrm{SNR}_{\mathrm{Gap}} = 0$ dB.

Fig. 16. Throughput comparison at $\mathrm{SNR}_{\mathrm{Gap}} = 4$ dB.

and the one obtained from simulation results in Figs. 12 and 13. From the figure, we can observe that the simulation results are consistent with the analytical results. The $\mathrm{SNR}_{\mathrm{loss}}$ is almost a constant for a fixed $\mathrm{SNR}_{\mathrm{Gap}}$ and the difference between the analytical and simulation results is within 0.3 dB.

Figs. 15 and 16 compare the corresponding throughput of these schemes. From the figures, we can note that the throughput

of the DTC-SIR is higher than both the DTCs with and without ARQ.

As shown in (54), the $\mathrm{SNR}_{\mathrm{Gap}}$ is determined by the power amplification factor ρ_P^{sr} and the geometrical distribution factor ρ_d^{sr}. In order to increase the $\mathrm{SNR}_{\mathrm{Gap}}$ enabling the DTC-SIR to approach the perfect DTC, we should increase ρ_P^{sr} and ρ_d^{sr}. This can be achieved by placing the relay closer to the source than to the destination and/or making the transmit power from the source larger than that from the relay.

VII. CONCLUSION

In this paper, we present a new DTC scheme. In the proposed scheme, the relay transmits the codeword soft information. Analytical results demonstrate that imperfect decoding at the relay has only a slight effect on the system performance and that the proposed scheme approaches the performance of perfect DTC. Simulation results confirm that the proposed scheme also approaches the outage probability bound of a distributed two-hop relay network at high SNR.

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Branka Vucetic (M'83–SM'00–F'03) received the B.S.E.E., M.S.E.E., and Ph.D. degrees in electrical engineering from the University of Belgrade, Belgrade, Yugoslavia, in 1972, 1978, and 1982, respectively.

During her career, she has held various research and academic positions in Yugoslavia, Australia, and the U.K. Since 1986, she has been with the Sydney University School of Electrical and Information Engineering, Sydney, Australia. She is currently the Director of The Telecommunications Laboratory, Sydney University. In the past decade, she has been working on a number of industry sponsored projects in wireless communications and mobile Internet. She has taught a wide range of undergraduate, postgraduate, and continuing education courses worldwide. She coauthored four books and more than 200 papers in telecommunications journals and conference proceedings. Her research interests include wireless communications, digital communication theory, coding, and multiuser detection.
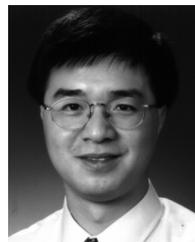
Tan F. Wong (SM'03) received the B.Sc. degree (First Class Honors) in electronic engineering from the Chinese University of Hong Kong, in 1991, and the M.S.E.E. and Ph.D. degrees in electrical engineering from Purdue University, West Layfayette, IN, in 1992 and 1997, respectively.

He was a Research Engineer in the Department of Electronics, Macquarie University, Sydney, Australia. He also served as a Postdoctoral Research Associate in the School of Electrical and Computer Engineering, Purdue University. Since August 1998, he has been with the University of Florida, where he is currently an Associate Professor of Electrical and Computer Engineering.

Dr. Wong serves as Editor for Wideband and Multiple-Access Wireless Systems for the IEEE TRANSACTIONS ON COMMUNICATIONS, and as the Editor for the IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY.

Yonghui Li (M'04) received the Ph.D. degree in electronic engineering from Beijing University of Aeronautics and Astronautics, Beijing, China, in November 2002.

From 1999 to 2003, he was affiliated with Linkair Communication, Inc., where he was a Project Manager with responsibility for the design of physical layer solutions for LAS-CDMA system. Since 2003, he has been with the Telecommunication Laboratory, University of Sydney, Sydney, Australia. He is a Lecturer in the School of Electrical and Information Engineering, University of Sydney. He serves as Associate Editor for *EURASIP Journal on Wireless Communications and Networking* and the *Journal of Networks*. His current research interests are in the area of wireless communications, with a particular focus on MIMO, multiple-user communications, coding techniques, and wireless sensor networks. He holds a number of patents granted and pending in these fields.

Dr. Li was awarded the Best Ph.D. Thesis Award. He has also been involved in the technical committee of several international conferences, such as ICC'05, PIMRC'05, WirelessCom'05, and so on.

Mischa Dohler (S'01–M'03) received the M.Sc. degree in telecommunications from King's College London, London, U.K., in 1999, the Diploma degree in electrical engineering from Dresden University of Technology, Dresden, Germany, in 2000, and the Ph.D. degree from King's College London, in 2003.

He has been a Lecturer at the Centre for Telecommunications Research, King's College London, until June 2005. He is now a Senior Research Expert in in the R&D Department, France Telecom, working on embedded and future communication systems, such as opportunistic radio, cognitive radio, and wireless sensor networks. Prior to telecommunications, he studied physics in Moscow. He has won various competitions in Mathematics and Physics, and participated in the third round of the International Physics Olympics for Germany. He has published over 50 technical journal and conference papers, holds several patents, coedited and contributed to several books, and has given numerous international short courses.

Dr. Dohler has been the Student Representative of the IEEE UKRI Section, member of the Student Activity Committee of IEEE Region 8, and the London Technology Network Business Fellow for King's College London. He has been TPC member and co-chair of various conferences. He is Editor for the *EURASIP Journal*, the IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY, and the IEEE COMMUNICATIONS LETTERS.