Superposition Coding in the Downlink of CDMA Cellular Systems

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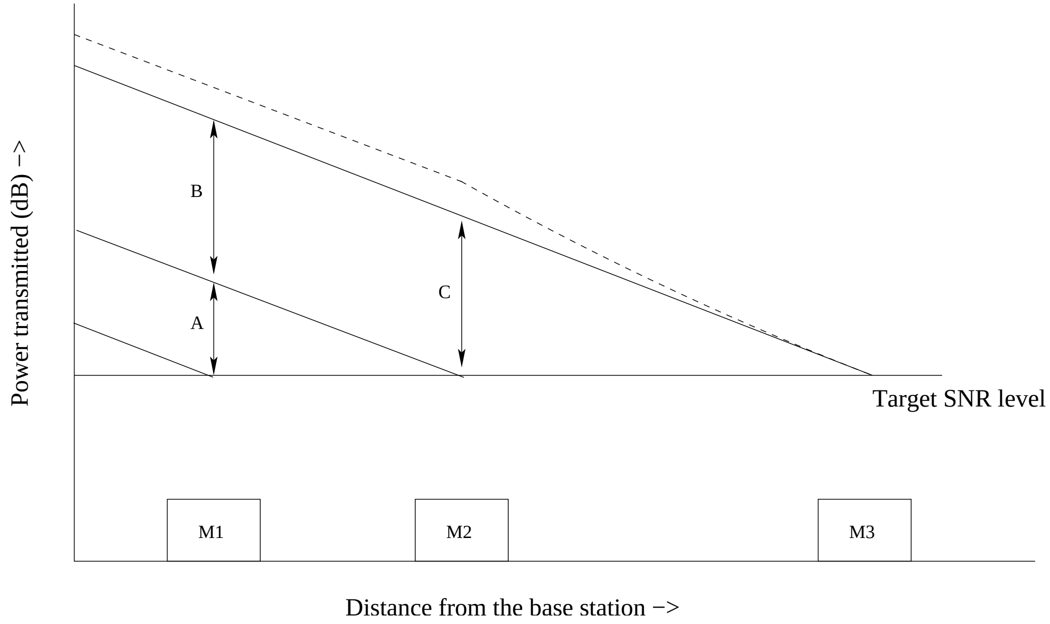
Outline of the talk

- Introduction and Motivation
- System Description
- User Capacity under Average Power Constraint
- Results and Discussion
- Conclusions

Introduction

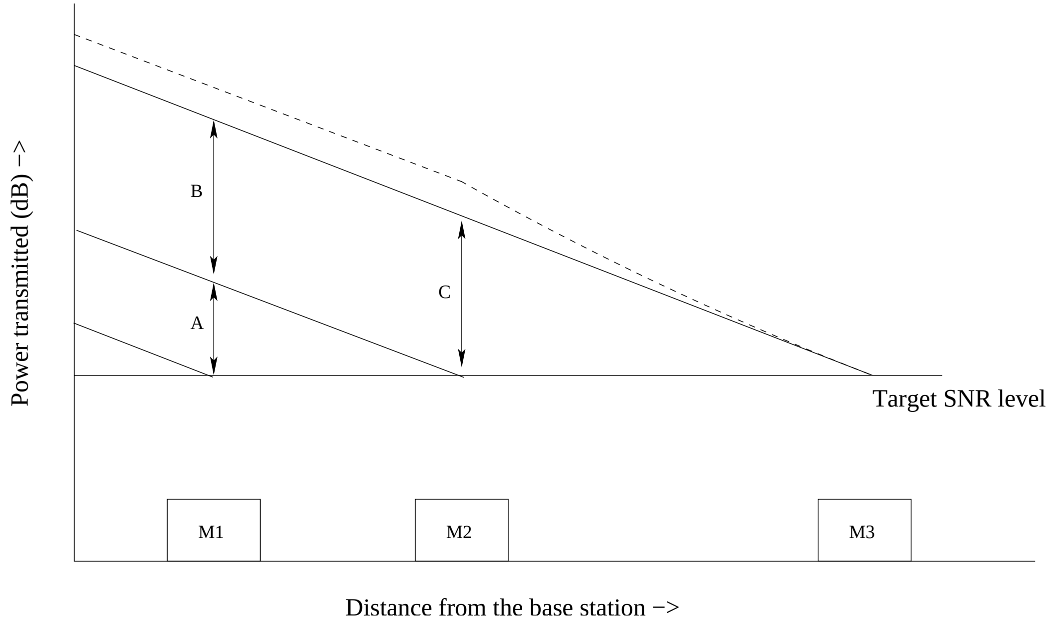
- Fading induces great disparity in the channel gains of radios in a CDMA cellular network.
- Power control is employed at the base station to maintain a constant SNR at the mobile radios.
- Ideally, each mobile radio sees the same SNR on their spreading code.
- If a radio despreads another radio's signal, it might receive the signal at much different SNR.
- Superposition coding offers significant advantages by transmitting to multiple users on a single spreading code.

Motivating Example



- Base station at the origin and radios M_1, M_2, M_3 arranged in decreasing order of channel gains.

Motivating Example



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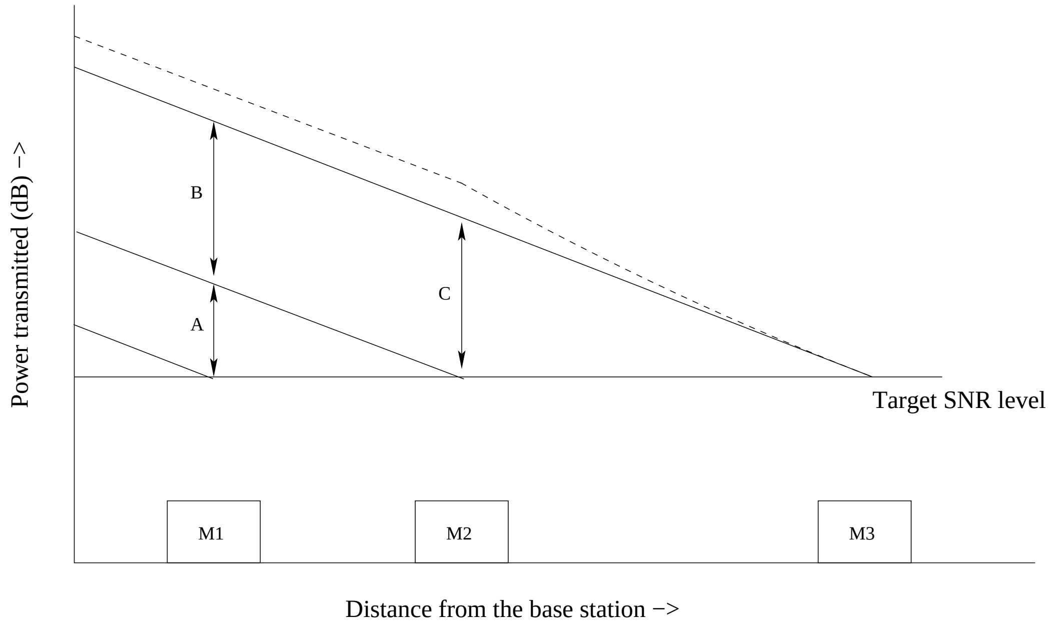
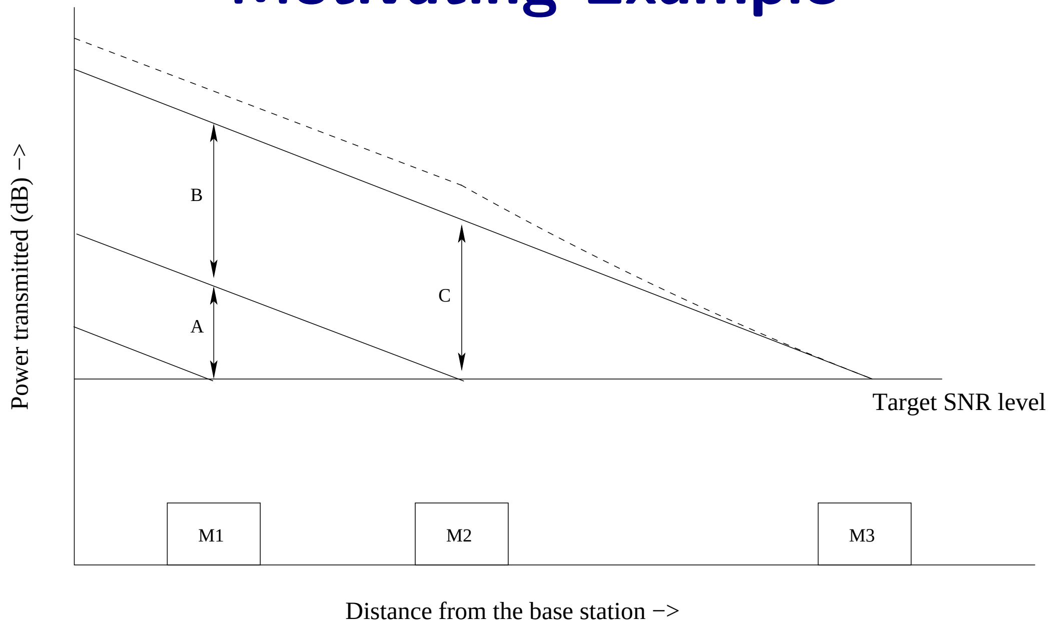
- Exponential path-loss channel with no fading. $(P_r \propto {d_r}^{-\alpha})$

Motivating Example



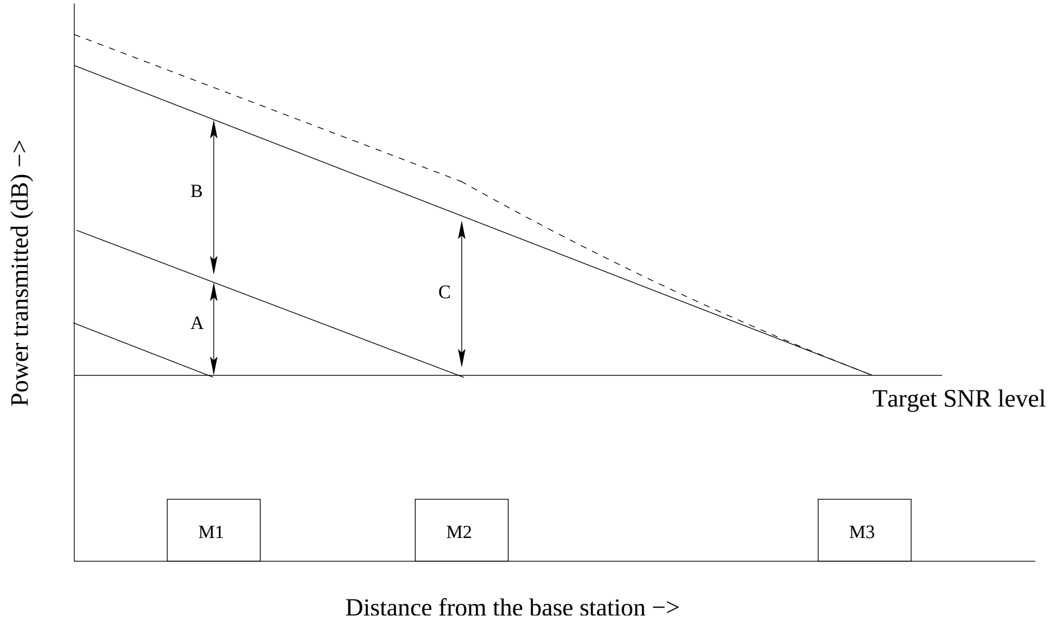
- Base station at the origin and radios M_1, M_2, M_3 arranged in decreasing order of channel gains.
- Exponential path-loss channel with no fading. $(P_r \propto {d_r}^{-\alpha})$
- Ordinate indicates the power transmitted by the base station to maintain the same target SNR at the radios.

Motivating Example



- When base station transmits to M_3, M_2 sees an additional CdB of power above its target SNR.
- Similarly, M_1 sees AdB of additional power when it decodes the signal intended for M_2.
- Superposition coding can be employed to achieve higher throughout or equivalently support more radios.

Motivating Example



- Superposition coding increases the total transmit power and hence the interference.

System Description

- Base station is at the center of a circular area of coverage.
- The radios are uniformly distributed in the area of coverage.
- The channel is modeled as an exponential path-loss channel with Rayleigh flat fading.

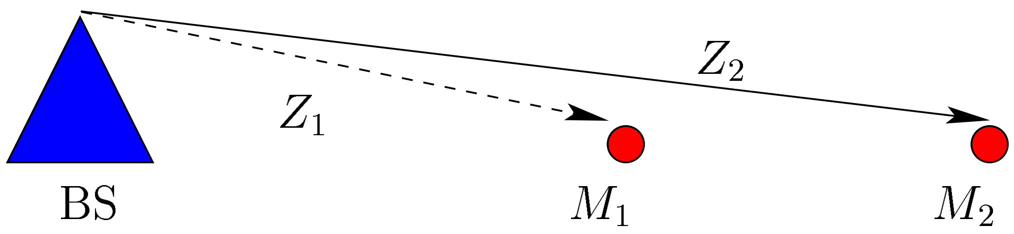
$$P_r = K_p d_r^{-\alpha} |h_r|^2 P_t$$

- The bandwidth seen by each radio after despreading is W Hz.
- All the radios have a common target SNR of γ dB.
- The number of orthogonal channels available is N.

System Description

Basic Message: The message with lower SNR requirement for its accurate reception is called *Basic Message*.

*Additional Message:*The message with higher SNR requirement for its accurate reception is called *Additional Message.*



$$R_{am} = W \log_2 \left(1 + \frac{aK_p Z_1 P}{N_0 W}\right) \quad (1)$$

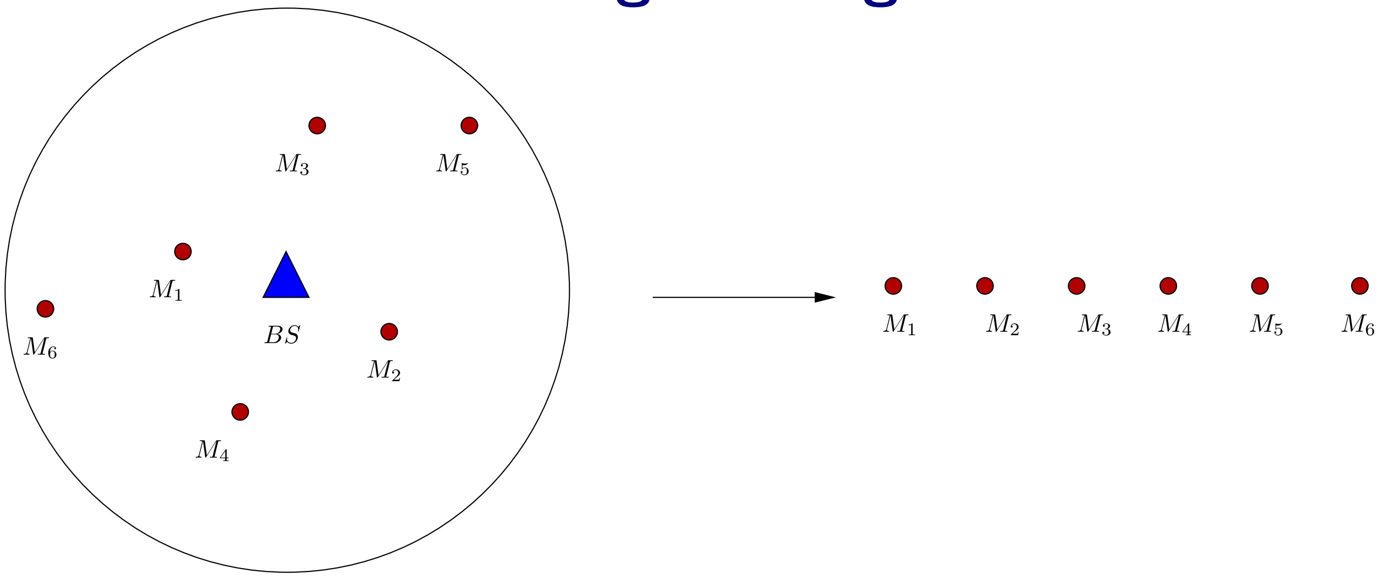
$$R_{bm} = W \log_2 \left(1 + \frac{(1-a)K_p Z_2 P}{aK_p Z_2 P + N_0 W}\right) \quad (2)$$

User Capacity

User Capacity: Number of users supported by the base station under an average transmit power constraint.

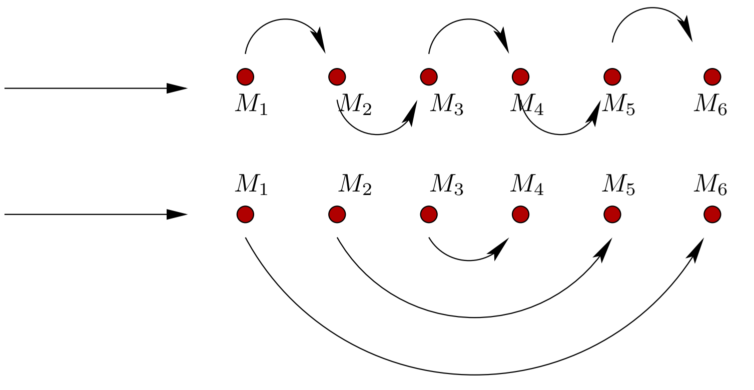
- The downlink user capacity under superposition coding depends on *pairs* of radios involved in superposition coding.
- *Pairing Strategy:* Let the radios be indexed in the decreasing order of channel gains. Pairing strategy $f(i)$ is a one-to-one function which pairs radio M_i with radio $M_{f(i)}, f(i) > i$, for $1 \leq i \leq N$. This implies that radios M_i and $M_{f(i)}$ share the same spreading code and M_i pairs with $M_{f(i)}$ to recover an additional message superimposed on the message for $M_{f(i)}$.

Pairing Strategies



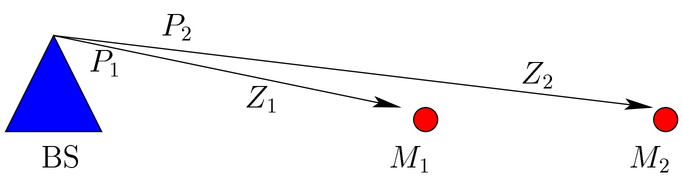
$f(i) = i + 1$

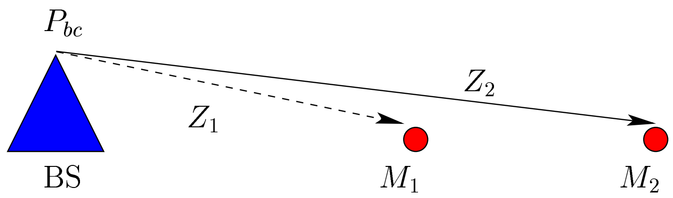
$f(i) = N + 1 - i$



Maximizing User Capacity

Proposition 1. *Consider a cellular network with K radios and N orthogonal channels such that $N < K \leq 2N$. The total transmitted power by the base station using N orthogonal channels and two-level superposition coding is greater than that of direct transmission to the K radios through K orthogonal channels.*

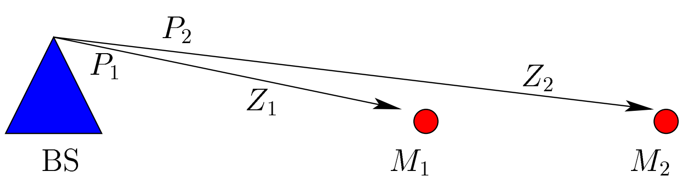


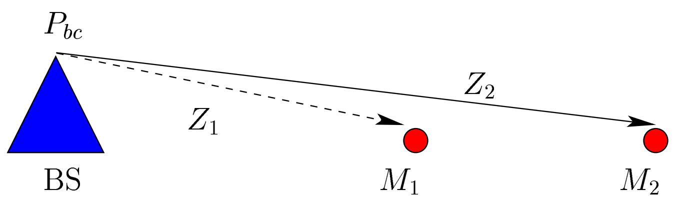


$$P_{bc} > P_1 + P_2$$

Maximizing User Capacity

Corollary 1. *The minimum additional power required for broadcasting to a pair of radios having the same spreading sequence is* γP_i*, where* γ *is the common target SNR and* P_i *is the power required by the base station to maintain a constant SNR of* γ *at the radio* M_i *with better channel gain and without employing broadcasting.*





$$P_{bc} = P_1 + P_2 + \gamma P_1$$

Maximizing User Capacity

Corollary. *A pairing strategy which minimizes the total transmitted power for a given number of pairs $k \leq N$ is*

$$f(i) = i + N, 1 \leq i \leq k.$$

The choice of the optimum pairing strategy is not unique, but the minimum total transmitted power is unique.

User capacity under average power constraint

- Compare the user capacity of a system employing superposition coding and the optimum pairing strategy to that of a system employing GWBE sequences under an average power constraint.
- Generalized Welch Bound Equality (GWBE) sequences are employed to support more radios than the processing gain of the network.
- We derive the average power constraint from a CDMA system supporting N radios through N orthogonal channels.
- Path-loss exponent $\alpha = 2$, for sake for analysis.

Cellular Network without superposition coding

- Cellular network with infinite population and N orthogonal channels.
- Radios are uniformly distributed in the circular area of coverage with unit radius.
- All the radios have target SNR requirement γ & outage probability of ρ.
- The distribution of the channel gain z of a radio is given by

$$\begin{aligned} F_Z(z) &= F_Z(z = d^{-2}|h|^2) \\ &= 1 + \frac{e^{-z} - 1}{z}, \; z > 0 \end{aligned}$$

Cellular Network without superposition coding

- An outage event occurs if the instantaneous SNR of the radio falls below γ, i.e. $\frac{K_p z P_t}{N_0 W} < \gamma$.

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- When an outage occurs, the base station doesn't transmit to that particular radio.
- Under infinite population assumption, it is always possible to find N radios with channels gains $z > Z_\rho$, where Z_ρ is the maximum value of channel gain that results in an outage.
- The average power transmitted by the base station to the N radios with $z \geq Z_\rho$ is

$$N\mathbb{E}_{P_T(Z_\rho)} = N\gamma N_0 W (K_p)^{-1} \left[\frac{1 + Z_\rho^2 \Gamma(0, Z_\rho) - e^{-Z_\rho}(1 + Z_p)}{2Z_\rho(1 - e^{-Z_\rho})} \right]$$

Cellular Network with superposition coding

- The base station transmits to K radios in every transmission interval.
- All the radios have a common target SNR of $\gamma^{'}$.
- Under infinite population, assumption we can find K radios with $z \geq Z_\rho$.
- The total power transmitted to K radios through N orthogonal channels using superposition coding is

$$\begin{aligned} P_T^{bc} &= \frac{\gamma^{'} N_0 W}{K_p} \left(\sum_{k=1}^{K} \frac{1}{z_k} + \gamma^{'} \sum_{k=1}^{K-N} \frac{1}{z_k} \right), \; z_1 > z_2 > \cdots > z_K \\ &= P_T^{nbc} + \Delta P_T^{bc} \end{aligned}$$

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&= P_T^{nbc} + \Delta P_T^{bc}
\end{aligned}
$$

- P_T^{nbc} can be interpreted as total power required to transmit to K users using K orthogonal codes (and target SNR $\gamma^{'}$).
- ΔP_T^{bc} can be interpreted as the increase in the transmitted power due to employing superposition coding to support K radios through N codes.

Cellular Network with superposition coding

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&= P_T^{nbc} + \Delta P_T^{bc}
\end{aligned}
$$

- The average total power transmitted to K radios is

$$
\mathbb{E}\{P_T^{bc}\} = \mathbb{E}\{P_T^{nbc}\} + \mathbb{E}\{\Delta P_T^{bc}\}
$$

Cellular Network employing GWBE sequences

- The base station transmits to K_g radios in every transmission interval.
- All the radios have a common target SNR of $\gamma^{'}$ and the processing gain of the system is N.
- Under infinite population, assumption we can find K_g radios with $z \geq Z_\rho$.
- The total power transmitted to K_g radios using GWBE sequences is given by

$$P_T^g = \frac{N g(\gamma^{'}) N_0 W / K_p}{N - K g(\gamma')} \sum_{k=1}^{K_g} \frac{1}{z_k}, \quad g(\gamma^{'}) = \frac{\gamma^{'}}{1 + \gamma'}$$

Cellular Network employing GWBE sequences

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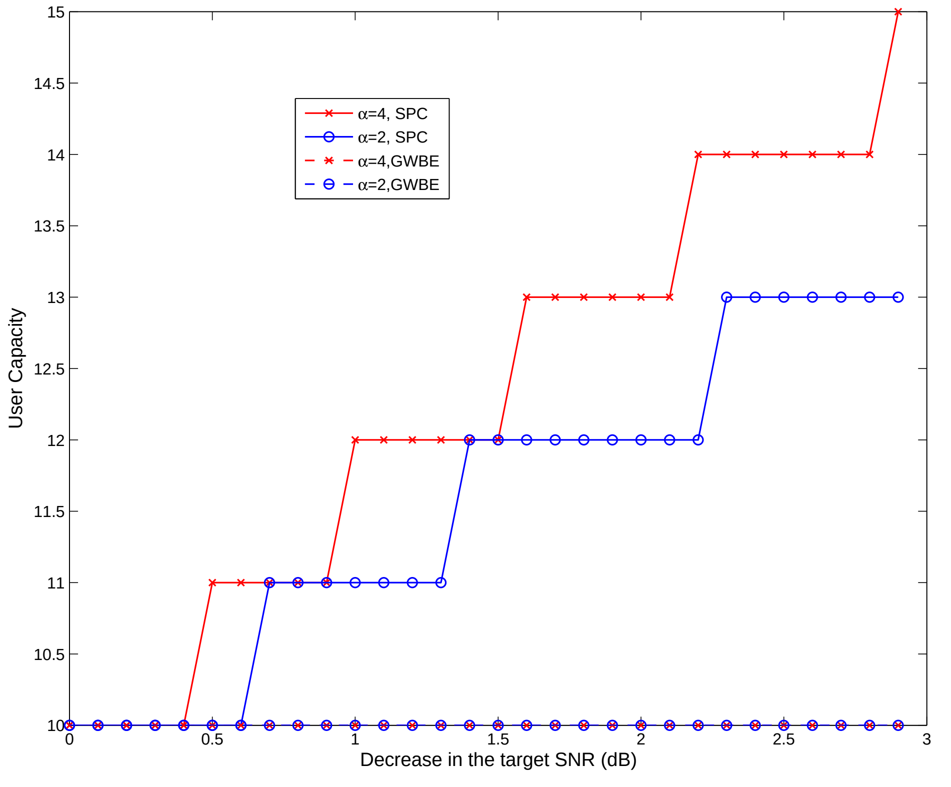
- $g(\gamma^{'})$ is called the effective bandwidth of the user.
- K_g is upper bounded by

$$K_g < N(1 + \frac{1}{\gamma^{'}})$$

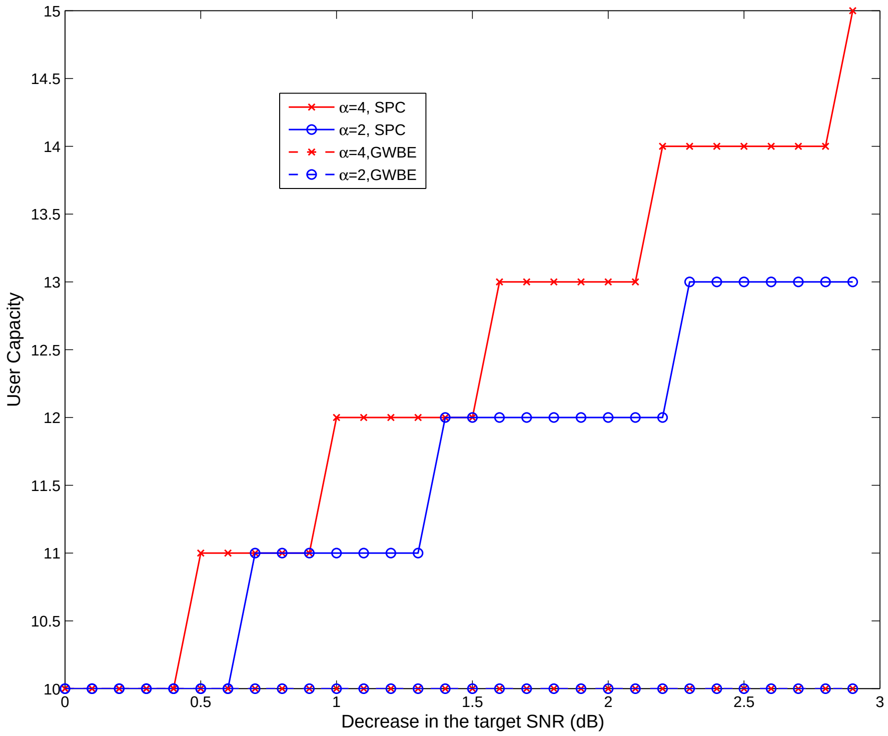
Results and Discussion

- Compare the user capacities K and K_g of systems employing superposition coding and GWBE sequences respectively, under the same average total power constraint.



Results and Discussion

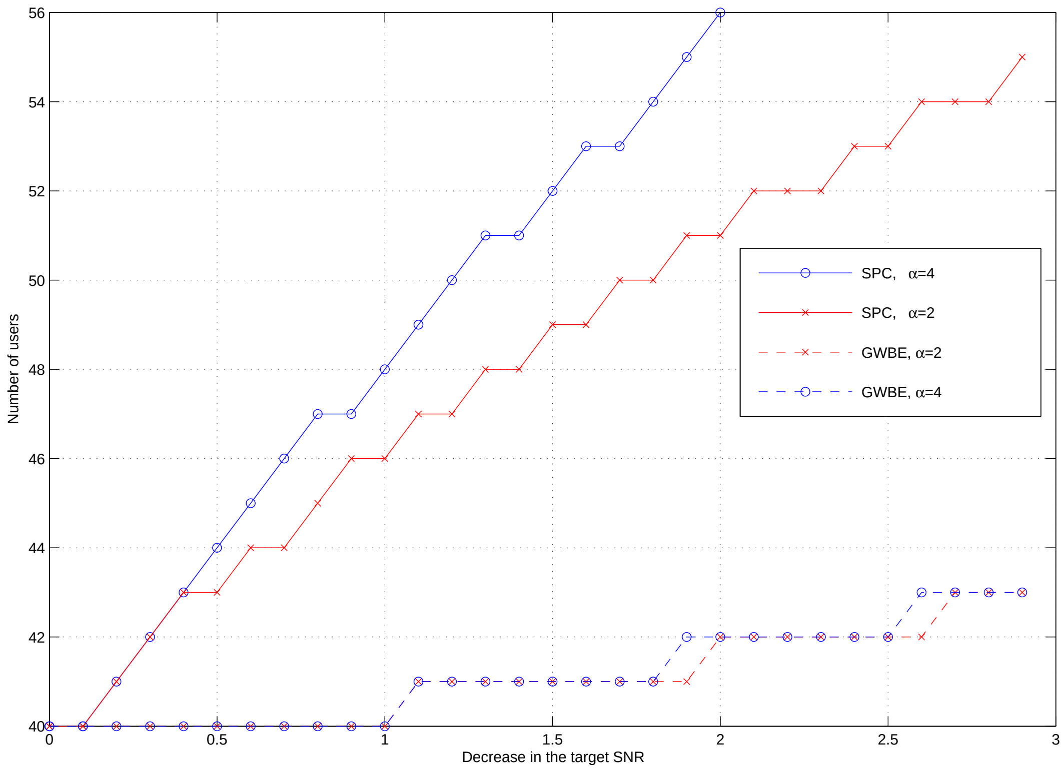
- $N = 10, \gamma = 10$dB, $N_0 = 10^{-10}$W/Hz, $W = 10^6$Hz, $\rho = 0.05$.
- User capacities plotted as a function of the degradation in the target SNR, $-10 \log \frac{\gamma'}{\gamma}$



Results and Discussion

- Superposition coding supports 10% more users for $\alpha = 2$ and 20% more users for $\alpha = 4$ compared to a conventional CDMA system and for a degradation of 1dB.
- Increase in the path-loss exponent increases the user capacity under superposition coding.
- GWBE sequences do not offer any advantage in this particular scenario.

Results and Discussion

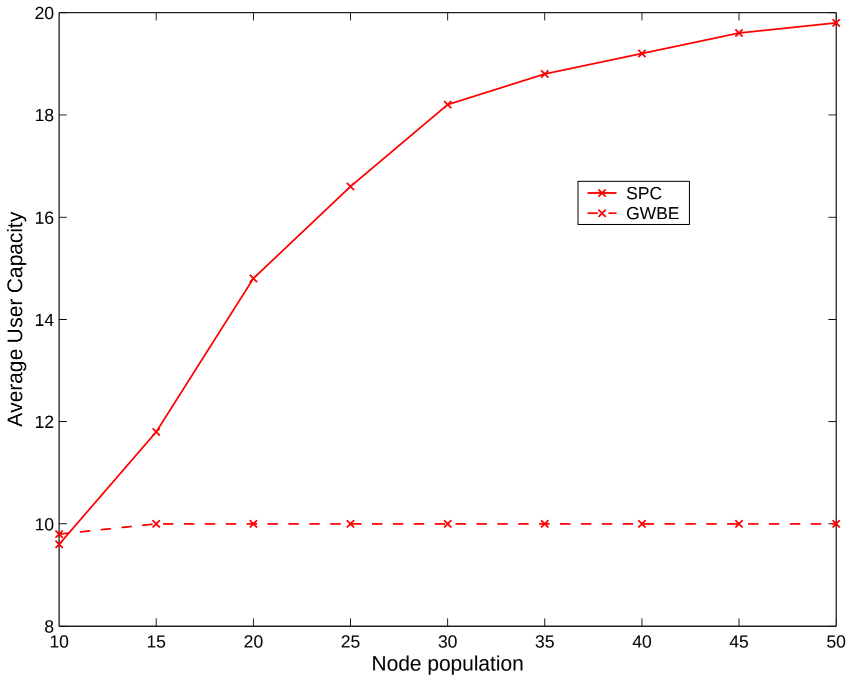


Similar trend is observed for N=40

User capacity under total power constraint

- Evaluate the average user capacity under finite radio assumption and total power constraint in a transmission interval.
- Comparison of the average user capacity of systems employing superposition coding and GWBE sequences under the same total power constraint.
- $N = 10$, $N_0 = 10^{-10}$, $W = 10^6$Hz, $\gamma^{'} = 10$dB.
- The total power constraint is arbitrarily chosen to be equal to the average power constraint considered earlier.

User capacity under total power constraint



- Superposition coding achieves $2N$ user capacity when the radio population is about 5 times the number of orthogonal channels available.
- GWBE sequences do not provide any additional gain.

Conclusions

- Evaluated the performance of superposition coding in increasing the user capacity of the forward link of CDMA cellular systems.
- Results indicate that on average 20% increase in the user capacity is possible for $\alpha = 4$ under an average power constraint and a degradation of 1dB in the taget SNR.
- With a fixed power constraint and finite radio population, the increase in the user capacity due to superposition coding is far greater than that of a system employing GWBE sequences.

Thank You