Best Permutation Search Strategy for Ultra-Wideband Signal Acquisition

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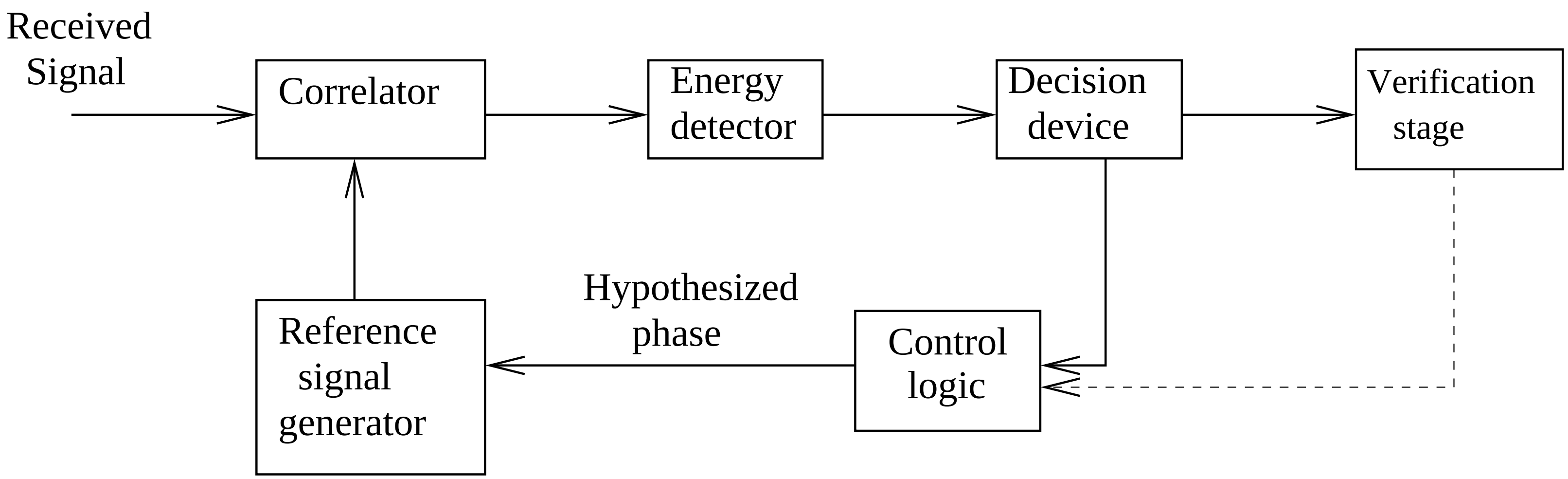
What is Acquisition?

- Timing Acquisition : Process of synchronizing the receiver clock to the transmitter clock

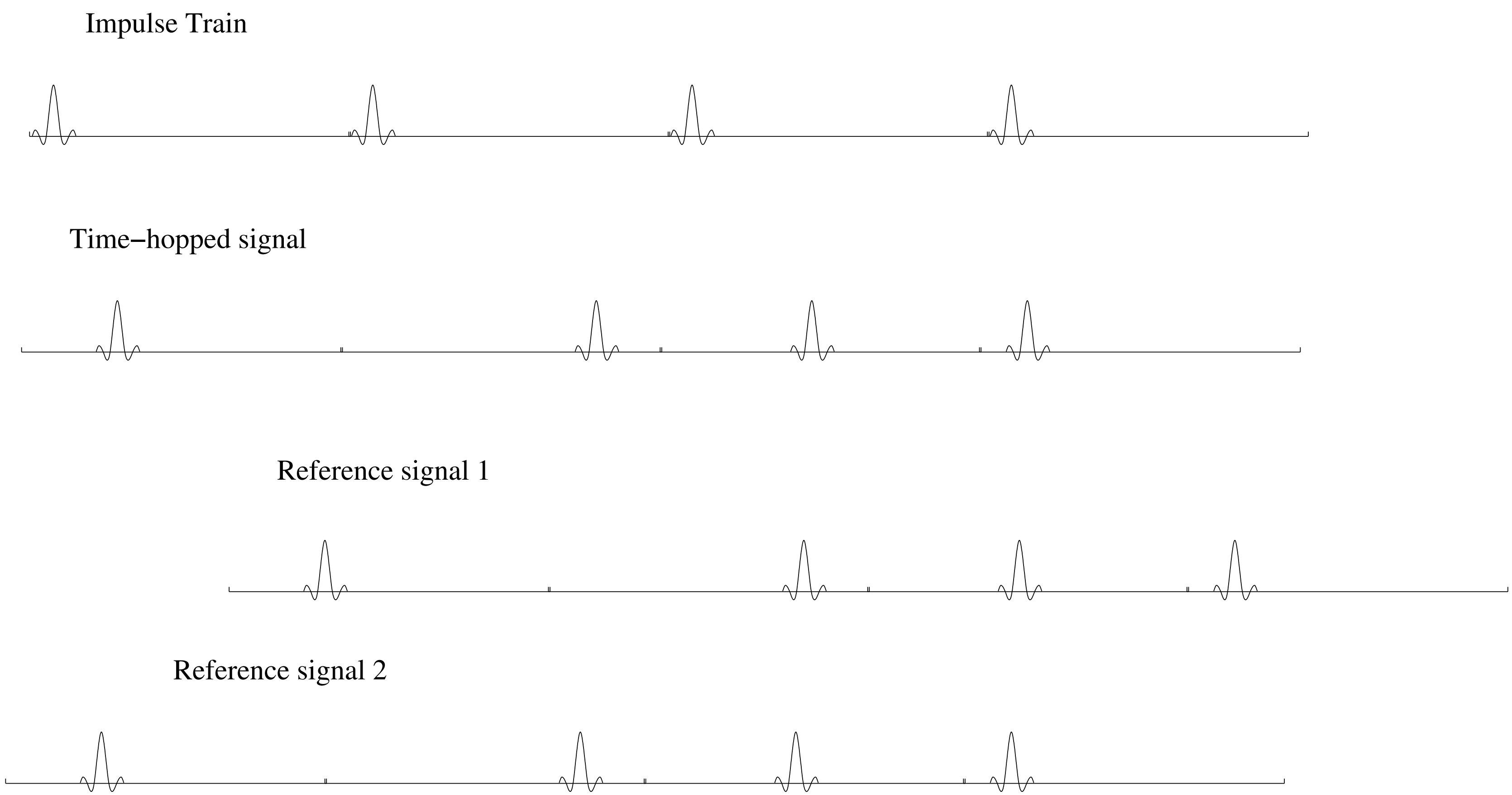
- Crucial for system performance in digital communications

Typical Acquisition System



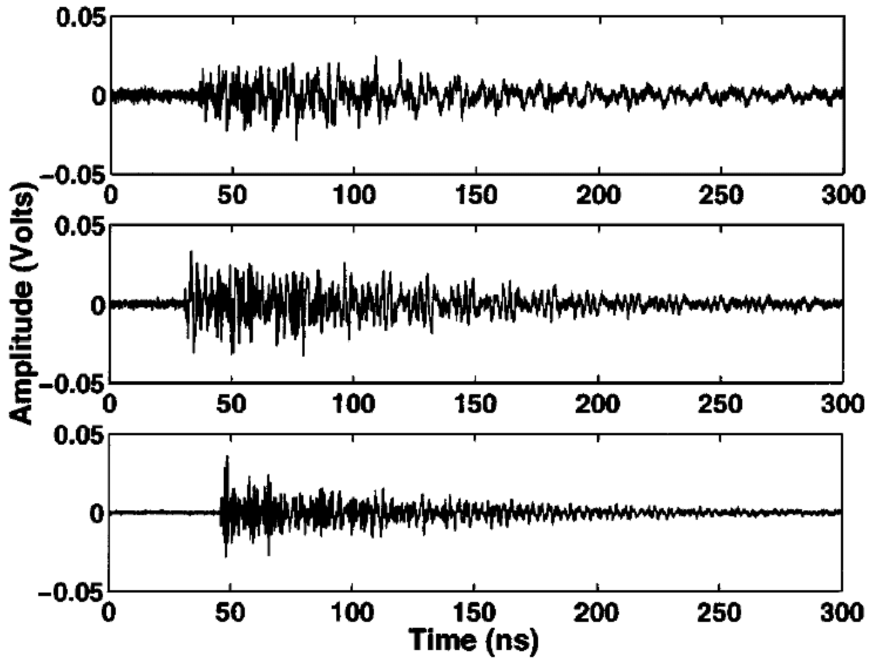
- Transmitter sends known periodic signal
- Receiver tries to guess the timing/phase of the received signal
- Noise causes *misses* and *false alarms*

Typical Acquisition System



The Ultra-Wideband Channel

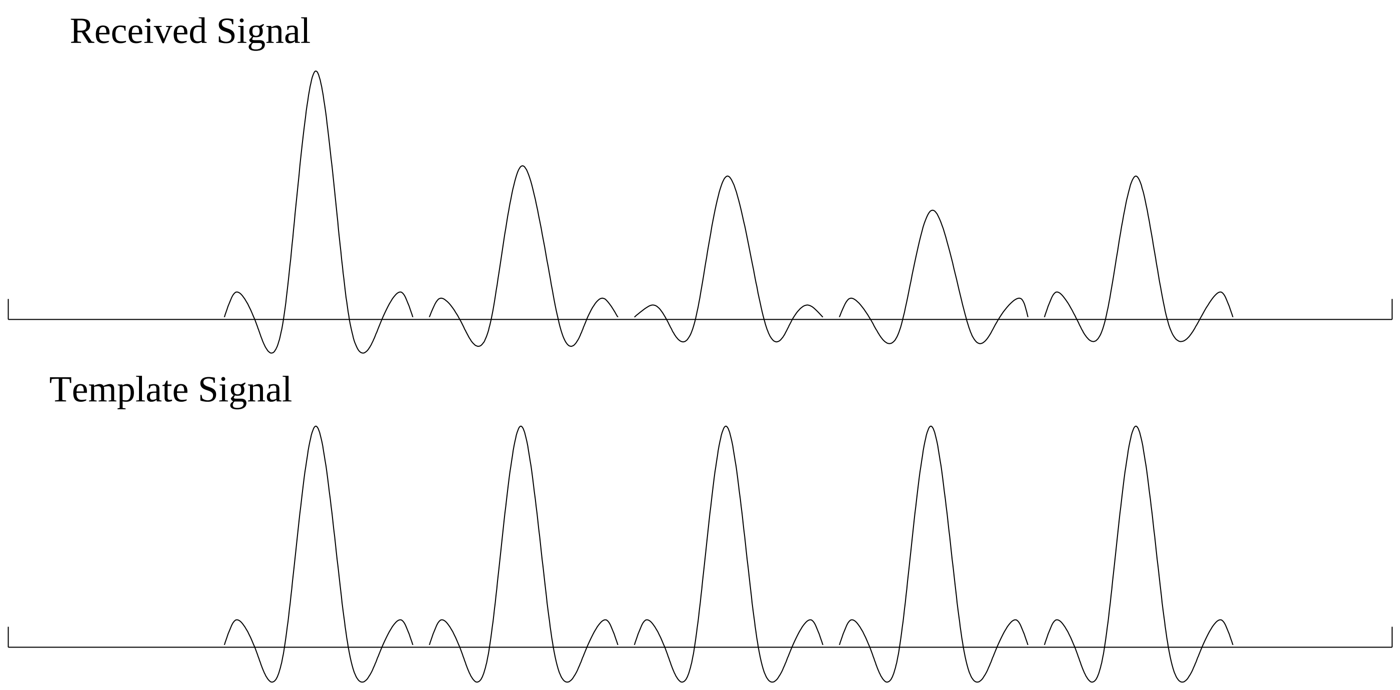
- UWB channel is a dense multipath channel



- Considerable amount of energy available in multipath components

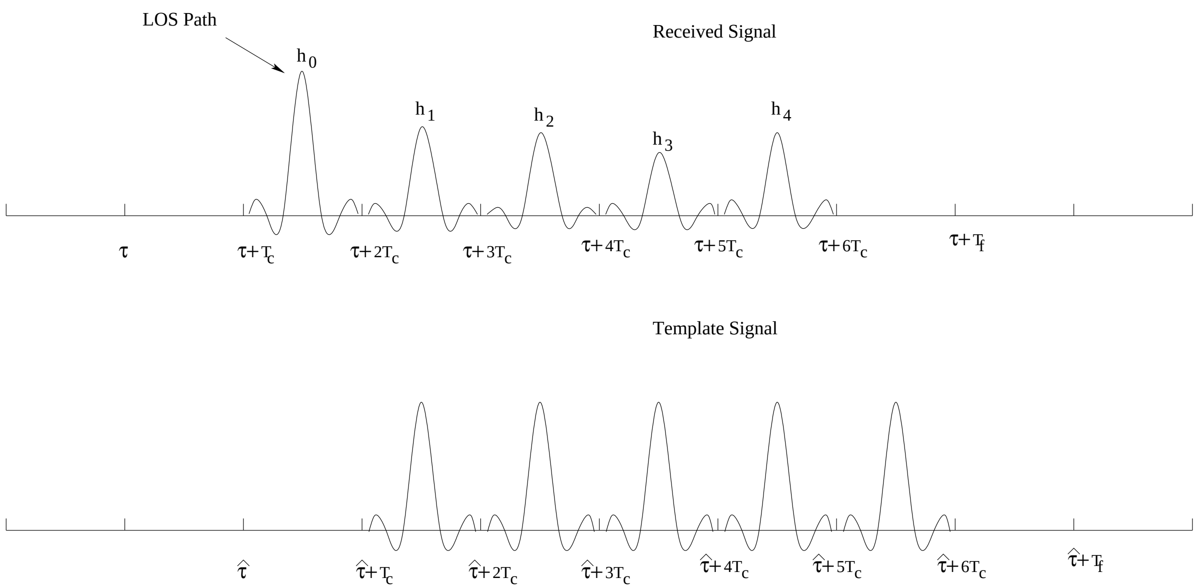
Equal Gain Combining

- Equal gain combining : Practical method to utilize energy in the multipaths



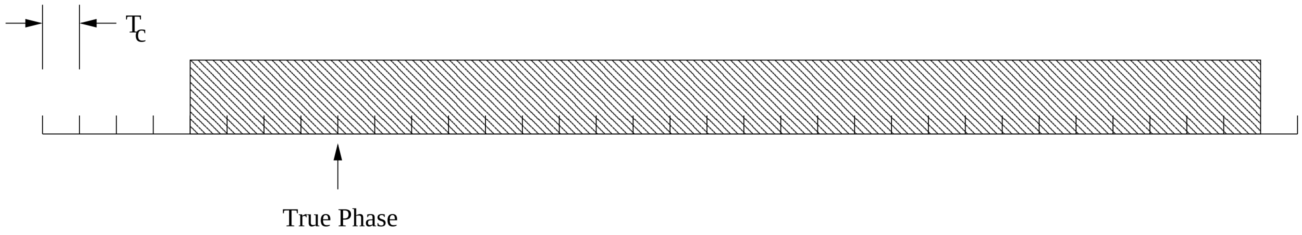
Key Features

- Receiver need not lock onto the LOS path to perform adequately

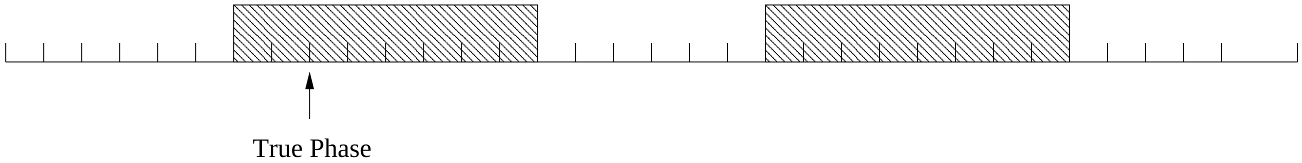


- Significant reduction in mean acquisition time

Typical Hit Sets

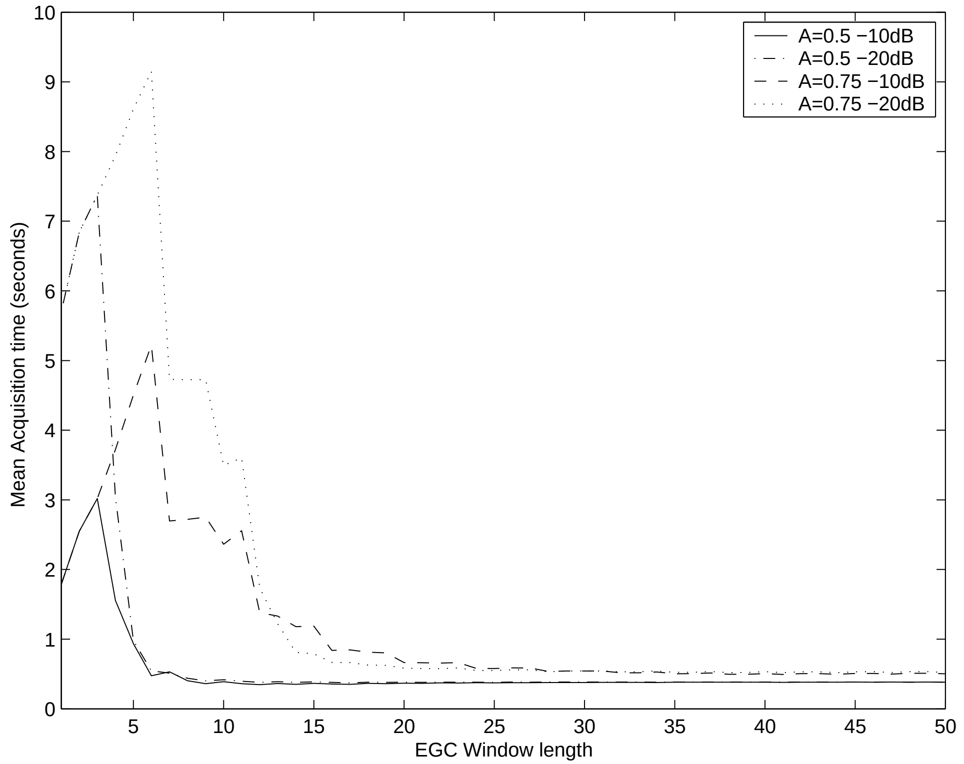


(a)

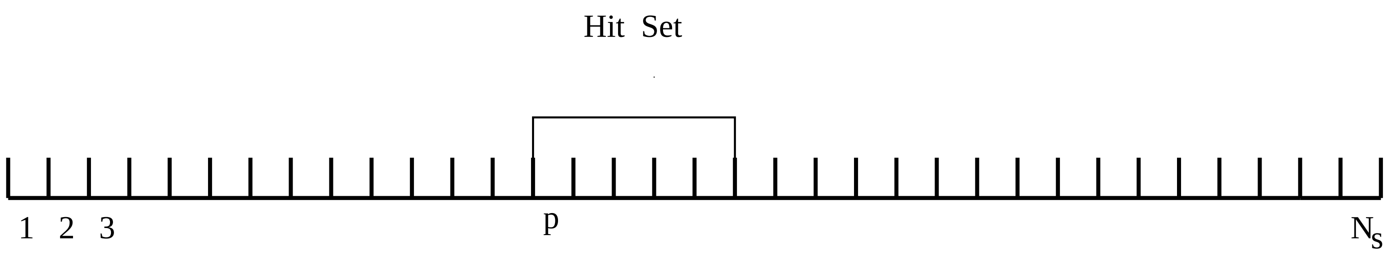


(b)

Mean Acquisition Time vs. EGC window length



What is the best search strategy?



- Serial search is no longer optimal in the case of a multiple hit set
- Mean acquisition time is chosen as the metric
- Restrict our attention to permutations of the search space
- In the low SNR region, the probabilities of detection of all the hit set elements are very close
- Reasonable assumption: They are all equal

System Model

- Search space can be represented by $S_p = \{1, 2, \ldots, N_s\}$
- The hit set S_h is H consecutive phases in the search space
- p is the position of the first element of the hit set
- P_d is the probability of detection in a hit set phase
- P_{fa} is the probability of false alarm in a non-hit set phase
- T is the dwell time and T_{fa} is the false alarm penalty time

Example

- Consider the case when $N_s = 8$ and $H = 3$.
- Let the permutation be $R = (1, 4, 7, 2, 5, 8, 3, 6)$

1	**2**	**3**	4	5	6	7	8
1	**2**	**3**	**4**	5	6	7	8
1	2	**3**	**4**	**5**	6	7	8
1	2	3	**4**	**5**	**6**	7	8
1	2	3	4	**5**	**6**	**7**	8
1	2	3	4	5	**6**	**7**	**8**
1	2	3	4	5	6	**7**	**8**
1	**2**	3	4	5	6	7	**8**

1	4	7	**2**	5	8	**3**	6
1	**4**	7	**2**	5	8	**3**	6
1	**4**	7	2	**5**	8	**3**	6
1	**4**	7	2	**5**	8	3	**6**
1	4	**7**	2	**5**	8	3	**6**
1	4	**7**	2	5	**8**	3	**6**
1	4	**7**	2	5	**8**	3	6
1	4	**7**	**2**	5	8	3	6

Mean Acquisition Time Calculation(1)

- Let $\{t_i(p) : i = 1, 2, \ldots, H\}$ are the positions of appearance of elements of hit set elements in the sequential search.
- Total acquisition time for a particular acquisition event is

$$T(p, i, j, k) = t_i(p)T + jN_sT + kT_{fa},$$

where an acquisition event is defined by the position $t_i(p)$ where we have a hit, a particular number of misses j of S_h, and a particular number of false alarms k in the elements evaluated which do not belong to S_h.

Mean Acquisition Time Calculation(2)

- The mean acquisition time conditioned on the fact that the first element of the hit set is in position p of the search space is given by

$$
\begin{aligned}
T_{acq}(p) &= \sum_{i=1}^{H}\sum_{j=0}^{\infty}\sum_{k=0}^{K} T(p,i,j,k)\binom{K}{k} P_{fa}^{k}(1-P_{fa})^{K-k} P_{M}^{j} P_{h}(i) \\
&= \frac{(T+P_{fa}T_{fa})\sum_{i=1}^{H} t_{pi}P_h(i)}{1-P_M} \\
&+ \frac{[N_sT+(N_s-H)P_{fa}T_{fa}]P_M - P_{fa}T_{fa}\sum_{i=1}^{H} iP_h(i)}{1-P_M}.
\end{aligned}
$$

Mean Acquisition Time Calculation(3)

- The mean acquisition time is then given by

$$\begin{aligned}\bar{T}_{acq} &= \frac{1}{N_s}\sum_{p=1}^{N_s} T_{acq}(p) \\ &= \frac{(T + P_{fa}T_{fa})\sum_{i=1}^{H}(\sum_{p=1}^{N_s} t_{pi})P_h(i)}{N_s(1-P_M)} \\ &+ \frac{[N_sT + (N_s - H)P_{fa}T_{fa}]P_M - P_{fa}T_{fa}\sum_{i=1}^{H} iP_h(i)}{1-P_M}.\end{aligned}$$

Minimizing the MAT

- We want to minimize
$$g(\mathbf{s}) = \sum_{i=1}^{H} s_i P_h(i),$$
where $\mathbf{s} = (s_H, s_{H-1}, \ldots, s_1)$ and $s_i = \sum_{p=1}^{N_s} t_{pi}$, over all permutations of S_p.

- $g(\mathbf{s})$ is Schur-concave on $\mathcal{A} = \{\mathbf{s} = (s_H, \ldots, s_1) : s_i = \sum_{p=1}^{N} t_{pi}, i = 1, 2, \ldots, H,$ for some permutation R of $S_p\}$.

Properties of Schur-concave functions

- We say that $\mathbf{x}$ is majorized by $\mathbf{y}$ or $\mathbf{x} \prec \mathbf{y}$ if and only if

$$\sum_{i=1}^{k} x_i \geq \sum_{i=1}^{k} y_i, \qquad k = 1, \ldots, H-1,$$

  and

$$\sum_{i=1}^{H} x_i = \sum_{i=1}^{H} y_i,$$

  where x_i, y_i are non-decreasing in i.

- If $\mathbf{x} \prec \mathbf{y}$, then $g(\mathbf{x}) \geq g(\mathbf{y})$.
- Thus the MAT is minimized if there is maximal vector in $\mathcal{A}$.

Finding the maximal vector

- A lower bound of $\sum_{i=1}^{k} s_i$ over all permutations of S_p is $\frac{N_k(N_k+1)H}{2} + (N_s k - N_k H)(N_k + 1)$ for $k = 1, \ldots, H$, where $N_k = \lfloor \frac{N_s k}{H} \rfloor$.
- This lower bound is achievable by a permutation $\mathbf{L}$ of S_p given by

$$L_i = (i-1)H \pmod{N_s} + \left\lfloor \frac{i-1}{(\frac{N_s}{h})} \right\rfloor + 1,$$

where L_i is the element in its ith position and h is the greatest common divisor of N_s and H.